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THE MATHEMATICS TEACHER

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THE MATHEMATICS TEACHER

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Devoted to the interests of mathematics in Elementary and Secondary Schools

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THE MATHEMATICS TEACHER

Volume XLIII Number 4

Edited by William David Reeve

The Present State of Evaluation of Critical Thinking in Algebra and Geometry[1]

By M. F. Rosskopf

Syracuse University, Syracuse, New York

Very early in the history of people much attention was given to the study of mathematics; practical applications alone made study necessary, but mathematical investigations were carried on, too, for their own sake. The secret society of the Pythagoreans (about 500 B. C.) set up a training school for the instruction of new members. The curriculum consisted of four subjects:—geometry, arithmetic, astronomy, and music. It was Plato (about the fourth century B. C.) who hung over his academic door a sign that read, "Let no one who is unacquainted with geometry enter here." The belief that a study of mathematics led to clear thinking and logical marshalling of ideas is evident from the titles of books popular in their day; there was Alcuin's (735–804) *Problems for Quickening the Mind* and Robert Recorde's (1510–1558) *The Whetstone of Witte*.

Evaluation was not a problem in those days. Only a small percentage of adults studied mathematics. The students were the clergy and the wealthy, enjoying study of mathematics and philosophy because of the order these two disciplines gave to their world. Members of both groups made as much progress as they were able; they had the leisure to pursue their studies and discussions as far as knowledge extended. There were no examinations to face, only their colleagues to try to trip them up with embarrassing questions and paradoxes and doubts.

The gradual broadening of educational opportunities and a shift in the distribution of wealth brought with them problems. Centers of study organized as universities; standards of excellence in study had to be maintained. Very early the university assumed the responsibility for putting a stamp of approval on its product by conferring on it a degree. Requirements for degrees were necessary. Naturally, in order to assure itself the best of available material, the university set up entrance requirements. The curriculum became more and more complex; there was more and more competition among subjects for student time. This brings us up to date.

Inclusion of larger and larger numbers of children in public education had a profound effect upon the thinking of those responsible for that education. Two questions had to be answered. What would this experience in school do for all these children? How would the state and nation benefit, in order to justify the cost of this

[1] Presented at the ninth summer meeting of the National Council of Teachers of Mathematics, Denver, Colorado, August 29 to September 1, 1949.

education? It was mandatory that continual checks be made upon teaching and students. Evaluation grew into a study of what *is* in the curriculum and what *should* be. This was a natural outgrowth of a testing program that attempted to measure the attainment of education's objectives.

Every report on the teaching of mathematics that has appeared in the last fifty years has included the development of critical thinking among its stated objectives. Perhaps this objective is referred to as logical thinking, problem-solving, scientific thinking, reflective thinking, nature of proof, or clear thinking—but all deal with desirable changes in the student to be effected through the study of mathematics.

The first of two important, recent reports has this to say about the objective, "Especially at the high school level, principles of deductive thinking can be most effectively taught in connection with a well-organized, substantial subject such as geometry, which, being logical itself and free from personal prejudice, can serve as a yardstick."[2] And this to say about evaluation, ". . . the welfare of pupils, and the future of mathematics in the secondary schools all demand an ever deeper and broader study of instructional problems. In such study evaluation plays a fundamental role."[3]

The second of the two publications makes the following statement concerning the objective, ". . . the major role of mathematics in developing desirable characteristics of personality lies in the contribution it can make to growth in the abilities involved in reflective thinking, or problem-solving."[4] With respect to evaluation this report states, "Evaluation . . . is . . . essential if a sufficient body of unbiased data on these points is to be available."[5]

It is difficult to separate the existing tests into the two classes of algebra and geometry tests. Moreover, many of the published tests were made for other subject matter areas besides mathematics; they are mentioned below because the techniques used in their construction could be applied to the building of mathematics tests.

No achievement type test is included in the list because critical thinking includes more than the ability to solve a problem successfully. All the achievement tests that this writer examined failed to do more than set a series of problems for students to do. This is all right; such tests get at the specific mathematics objectives of teachers and school systems, but the evaluation techniques used in these achievement tests do not secure information concerning mathematics' contribution to the development of critical thinking.

The writer makes no claim for completeness of the list below. There are many tests on critical thinking, which have been devised by individual teachers and individual school systems, that have not been published. However, the list is the result of a careful check of references in periodicals, lists of published tests, and files of the Evaluation Service Center, Syracuse University.

The tests are listed in the order of their publication dates rather than by author. In that way a perspective of the development in construction techniques can be obtained that otherwise might be hidden. Without more apology or remarks here is the list of tests. This, then, is the present state of evaluation of critical thinking in algebra and geometry.

---

[2] *The Place of Mathematics in Secondary Education*, report of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics, p. 39. Fifteenth Yearbook of the National Council of Teachers of Mathematics. New York: Bureau of Publications, Teachers College, Columbia University, 1940.

[3] *Ibid.*, p. 186.

[4] *Mathematics in General Education*, Report of the Committee on the Function of Mathematics in General Education for the Commission on Secondary School Curriculum, p. 59. New York: D. Appleton-Century Co., 1940.

[5] *Ibid.*, p. 338.

Downing, Elliot, R. "Some Results of a Test of Scientific Thinking," *Science Education*, XX (October, 1936), 121–128.

A complete test on scientific thinking is given in this paper, together with scoring and administering directions. The test was given to students in grades eight through twelve and to college freshmen.

Arnold, Dwight L. "Testing Ability to Use Data in the 5th and 6th Grades," *Educational Research Bulletin*, XVII (December, 1938), 255–259.

The items in this test present problems in the relevancy and adequacy of data; other items obtain bias reactions of pupils to data presented.

Fawcett, Harold P. "The Nature of Proof," *Thirteenth Yearbook of the National Council of Teachers of Mathematics*. New York: Bureau of Publications, Teachers College, Columbia University, 1938.

In pages 123–126 there are presented a series of paragraphs culled from newspaper articles, editorials, and advertising. Students are directed to react to the paragraphs, judging whether or not the argument is logical, point out words that need definition, and state the assumptions made.

Test 2.51, Interpretation of Data. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1939.

The test presents statistical graphs and problem situations in science for interpretation.

Test 5.11, Application of Certain Principles of Logical Reasoning. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1939.

From three suggested conclusions following a paragraph description of a situation students are directed to choose one. Then, from a list of statements, those must be marked that support the conclusion chosen. The principles of logical reasoning tested are assuming the converse; in an indirect argument all possibilities must be considered (referred to later as the "indirect argument" principle); if the assumptions of an argument are accepted, then the conclusion that logically follows must be accepted (this principle will be referred to as the "if-then" principle); attacking the arguer does not attack his argument (referred to later as the "ad hominen" principle).

Test 5.21, Nature of Proof. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1939.

In this test a conclusion is presented with each test situation. From a list of statements the student must, first, select those that support and those that contradict the conclusion; second, select from these the statements that support the conclusion; and, third, select from the second group the statements that require more information.

Test 5.31, Analysis of Controversial Writing. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1939.

Quotations dealing with social questions are presented with a list of statements for each quotation. Students are directed to react to each statement on a three point scale:—agree, disagree, or no evidence.

CR 1, Exercises in Critical Reflection (For College Students). Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1939.

Test 5.12, Application of Certain Principles of Logical Thinking. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1940.

From among three suggested conclusions students are directed to choose one that seems to follow logically from the situation presented. Each supporting statement is checked as follows:—it is an explanation of why the conclusion chosen is logical; it is not an explanation of the conclusion; more information is necessary. The principles of logical reasoning tested are if-then, definition, ad hominen, indirect argument.

Test 5.22, Nature of Proof. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1940.

This test is a refinement of Test 5.21, above, with new situations.

Test 2.52, Interpretation of Data. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1940.

The test is a refinement of Test 2.51, with different problems.

Test 2.71, Interpretation of Data. Progressive Education Association, Evaluation in the Eight Year Study. Chicago: University of Chicago, 1940.

This is a junior high school test that presents statistical data for interpretation. Each supporting statement must be judged on a three point scale:—true, false, or not enough information given.

Test 5.2, Nature of Proof. Columbus, Ohio: Bureau of Education Research, The Ohio State University, 1940.

In a chemistry experiment students are directed to distinguish among facts, assumptions, and irrelevant statements.

Harding, L. W. *Value-Type Generalization Test.* Columbus, Ohio: College of Education, The Ohio State University, 1941.

The test presents a series of statements on social questions that students are directed to react to on a five-point scale:—strongly agree, agree, uncertain, disagree, strongly disagree.

Harding, L. W. *Value-Type Problemaire.* Columbus, Ohio: College of Education, The Ohio State University, 1941.

This is a test for teachers modeled after the preceding test.

Tyler, R. W. "Progress Report of the Evaluation Staff," *Progressive Education*, XVIII (January, 1941), 54–56.

The paper summarizes the work that had been done up to the date of publication and gives sample items from a variety of types of tests.

Smith, Eugene R., Tyler, Ralph W., et al, *Appraising and Recording Student Progress.* New York: Harpers Bros., 1942.

This book considers the basis for evaluation, presents an analysis of sample items from tests, and indicates methods of recording and interpreting the test data.

Watson, Goodwin, and Glaser, E. M. *Watson-Glaser Tests of Critical Thinking.* Yonkers, New York: World Book Co., 1942.

These tests have two forms, I-A and II-A. A problem situation with following statements is presented. The statements must be judged by the students as relevant or irrelevant to the problem situation.

Taba, Hilda. "The Evaluation of Critical Thinking," *Teaching Critical Thinking in the Social Studies*, Part Four, pp. 123–175. Thirteenth Yearbook of the National Council for the Social Studies, Washington, D. C.: National Council for the Social Studies, 1942.

The part that is referred to above presents a critique of evaluation of critical thinking and discusses various sorts of test items.

Foust, J. W., and Schorling, Raleigh. *Foust-Schorling Test of Functional Thinking in Mathematics.* Yonkers, New York: World Book Co., 1942.

There are two forms of this test, A and B. The manual of directions states that the test was constructed to fulfill ". . . the desire to handle quantitative problem situations in such a way in mathematics that these same situations will be more readily recognized, more fully appreciated, and more effectively handled when encountered outside the mathematics class in other fields of study or in life situations generally."

Nonverbal Symbolism. Chicago: Board of Examinations, University of Chicago, 1943.

This test is designed for college students. The test items introduce symbols of operation different from the usual ones, define them, and check on their use in problems.

Test of Logical Reasoning. Chicago: Board of Examinations, University of Chicago, 1943.

Certain statements are assumed and a conclusion is stated. From among five supporting statements the student is directed to choose one as the most intelligent comment on the conclusion.

Raths, Louis. "A Thinking Test," *Educational Research Bulletin*, XXIII (March, 1944), 72–75.

This is a test for the upper elementary grades, although it may be used in the junior high school. A story is presented; following statements check on a pupil's reaction to name-calling, authority, superstition, school loyalty, rationalizing, and going beyond the given data.

Test of Educational Progress in General Skills. Study of Educational Progress. Chicago: University of Chicago, 1946.

Statements and a conclusion are given, followed by five comments; students are directed to select the one statement that is the best comment on the conclusion. There are simple syllogisms and problems in symbolic algebra.

Test of Educational Progress in Mathematics. Study of Educational Progress. Chicago: University of Chicago, 1946.

Questions are presented that test a student's understanding of the fundamental operations of arithemetic and the basic concepts concerning the use of symbols in algebra. Problems involving the use of geometric concepts and principles appear also.

Hartung, Maurice L., and Fawcett, Harold P. "The Measurement of Understanding in Secondary-School Mathematics," *The Measurement of Understanding*, Part I, pp. 157–174. Forty-fifth Yearbook of the National Society for the Study of Education, Chicago: University of Chicago Press, 1946.

This chapter of the yearbook summarizes what has been done and takes a forward look at evaluation of understanding in mathematics. Many types of test items are suggested.

Harkman, F. C., and Ullsvik, Bjorne R. *Critical Thinking; Part I, Judging Conclusions; Part II, Meaning of Terms; Part III, Appraising the Evidence*. Unpublished, experimental tests. Normal, Illinois: Illinois State Normal University, 1948.

Following is a list of doctors' theses underway (or completed) as listed in the March issue of *Phi Delta Kappan* for the years 1947, 1948, and 1949. The titles of these theses seem to indicate that evaluation instruments will be constructed by the candidates. Following the name of the candidate and the title of his dissertation are the names of the university and the candidate's adviser.

Phi Delta Kappan, March, 1947.

Brune, Irvine H. *Semantics and Proof*. The Ohio State University, Fawcett.

Phi Delta Kappan, March, 1948.

Baker, James F. *Measuring Critical Thinking in Emotional Situations*. Boston University, Kvaraceus.

Charters, Alexander N. *An Evaluation of the Development of Thinking by Adult Education*. University of Chicago, Houle.

Chenoweth, Ralph W. *The Teaching of Critical Thinking in the Small High School*. University of Illinois, Smith.

Davis, David J. *The Preparation and Standardization of a Test in High School Mathematics*. University of Michigan, Woody.

Dane, John F. *Geometric Transfer: A Study in Ingenuity*. Columbia University, Gates, Clark, Walker, Thorndike.

Lakton, Robert. *Secondary Year Algebra Test*. University of Michigan, Schorling.

Phi Delta Kappan, March, 1949.

Alexander, Martha. *The Improvement in Critical Thinking Concomitant with a Year's Course in Tenth-Grade Mathematics*. University of Chicago, Buswell.

Edward, Thomas B. *Measurement of Some Aspects of Critical Thinking*. University of California (Berkeley), Parker.

Jackson, W. N. *The Role of Algebra in the Development of Relational Thinking*. The Ohio State University, Fawcett.

The list of published tests that concern themselves with aspects of critical thinking is not long. It is time for teachers of mathematics to concentrate some of their efforts on the problems of evaluation. All of us ought to cooperate with those teachers who are making an effort to perfect a test and to secure data for norms. Otherwise, as Hartung points out, "Without steady progress in evaluation, as well as in curriculum reform, adverse criticism of mathematics instruction, by colleges and the general public, will continue."[6]

In the discussion that followed the pres-

[6] Maurice L. Hartung, "A Forward Look at Evaluation," THE MATHEMATICS TEACHER, XLII (January, 1949), 33.

entation of this paper it was proposed that the writer serve as a clearing house for information on tests dealing with the development of critical thinking in algebra and geometry. From time to time, papers could be published, completing and bringing up to date the present bibliography.

The writer is quite willing to serve mathematics teaching in this way and will welcome correspondence giving references to existing tests overlooked in this bibliography, samples of test items, completed tests, or proposed tests that need to be given to students.

Institute for Teachers of Mathematics, Duke University

The tenth annual meeting of the Institute for Teachers of Mathematics will be held at Duke University, Durham, N. C., August 7–18.

The general Theme of the Institute is "Mathematics at Work."

There will be ten study groups

I. Advanced Geometry with Applications (High School and College)
Leader: Col. R. C. Yates

II. Mathematics in Grades 5 and 6
Leader: Mrs. Ida Mae Heard

III. Mathematics for Slow Students (Junior and Senior High School)
Leader: Miss Mary A. Potter

IV. Supervisors Discussion Group
Leader: Miss Veryl Schult

V. Aids in the Study of Algebra (Junior and Senior High School)
Leader: Miss Frances Johnson

VI. Graphical Analysis and Applications (High School and College)
Leader: Professor J. W. Cell

VII. Junior High School Mathematics
Leader: Mrs. Nanette R. Blackiston

VIII. Number Theory Made Elementary with Applications (High School and College)
Leader: Professor A. T. Brower

IX. Laboratory—Junior and Senior High School
Leaders: Miss Alene Archer, Miss Ida Mae Bernhard

X. Laboratory—High School and College
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Junior College Mathematics in View of the President's Report[1]

By HOUSTON T. KARNES[2]
Louisiana State University, Baton Rouge, Louisiana

MOST junior college teachers are conversant with the development of the junior college in this country, as well as the controversy with regard to its relationship to the secondary school on the one hand and to the senior college on the other. The junior college is relatively new and the debates have been frequent and long over its classification and the curriculum it should offer. Being new was not the only reason for the controversies; the fact of variation in type had a great deal to do with it. There was the so-called finishing school for women, the small church school, the state school serving as a feeder for its university, and the schools which were formed simply by adding two years to the existing secondary school program. This variation in type hindered the formulation of general curriculum patterns. Beyond this the junior college, formed by adding two years of work to a secondary school program, had the most difficult problems in its heterogeneous student body—that is, students who were planning to go on to a senior college and those who were ending their formal education with the fourteenth year.

All of this talk is old to most of us. For the others I suggest that you read Chapter 8 in the *Fifteenth Yearbook* of the National Council of Teachers of Mathematics as to the concepts which had been determined up to 1940.[3] Should no changes be made, we might continue as we have been doing, that is, for the most part, preparing the junior college student for the senior college even though most of them never transfer. (Of course this was never the ideal situation; nevertheless, it was followed for many reasons.) However, if the report of the President's Commission[4] is prophetic, it is time for us to study the problems of offerings and instruction again with a view to making some changes.

On page 39, Volume I of the *Report*,[5] the Commission predicts that in 1960 a minimum of 4,600,000 young people will be seeking education beyond the twelfth grade. Of this number 2,500,000 will be enrolled in junior colleges. In 1947 there were only 2,354,000 students in all colleges which is about half of the predicted enrollment for 1960. Each of us knows only too well how our facilities were crowded and of the teacher shortage during the past few years. Our problems begin to assert themselves. More junior colleges will have to be built and/or present facilities enlarged, more teachers will have to be procured, and the curriculum will have to be expanded. A quotation from the *Report*[6] will make us even more conscious of these problems.

> As we bring more and more students to the campus, we shall increase in proportion the tremendous variety of human and social needs the college programs must meet. We shall add to the already overwhelming diversity of aptitudes, interests, and levels of attainment that characterize the college student body. And so we shall have to increase the diversification of curricular offerings and of teaching methods and materials to correspond.

[1] "Higher Education for American Democracy" (A Report of the President's Commission on Higher Education) U. S. Government Printing Office: Washington, 1947.

[2] This paper was used as a basis for one of the discussion groups at the Denver meeting of the National Council of Teachers of Mathematics, August 29, to September 1, 1949.

[3] "The Place of Mathematics in Secondary Education," "Fifteenth Yearbook of the National Council of Teachers of Mathematics," Bureau of Publications, Teachers College, Columbia University, New York, 1940.

[4] "Higher Education for American Democracy," *op. cit.*

[5] *Ibid.*

[6] *Ibid.*, p. 44.

Yet in the midst of all the necessary diversity we must somehow preserve and expand a central unity. We must make sure that the education of every student includes the kind of learning and experience that is essential to fit free men to live in a free society.

The balance of this paper will be concerned with two matters: (1) the mathematical curriculum for the junior college in light of the above predictions, and (2) the procurement and training of teachers to administer the program. Some of the things which will be suggested are not new. The commission, whose work was published as the *Fifteenth Yearbook*,[7] recognized certain deficiencies in the mathematical curriculum and gave excellent council as to beneficial changes. However, because of the war and later crowded conditions, lack of teachers and insufficient funds, the mathematics courses in the junior college are, to a large degree, still patterned after the lower-division courses in the universities.[8] If our educational affairs in general are to be those which existed before the war, corrective changes may never take place. However, if the predictions come to pass, we shall, of necessity, have to change our curriculum. At present we do not know the fate of the Federal Aid to Education Bill. Nevertheless, if it is not passed during the present session of Congress, it will no doubt be enacted at some later date, since the N.E.A. and other agencies are strong supporters of the measure. When and if federal aid does come, those in charge of administering it will no doubt pay much attention to the *President's Report*. Therefore, we must begin to think of the offerings and those who will do the teaching at that level which will no doubt experience the greatest increase—the public junior college.

[7] "The Place of Mathematics in Secondary Education," *op. cit.*, Chapter VIII.

[8] This condition was true in 1940, *ibid.*, p. 153. The author knows of no similar statistical study since 1940. However, having been a consultant to the mathematics sections of two junior college conferences during the past year and a half, it is apparent that the conditions still exist.

Section I. The Curriculum

As has always been true there are two distinct classes of students in the junior college. Group 1, by far the larger, contains those who are terminating their formal education with the fourteenth year. The smaller class, Group 2, is composed of those who are going on to a senior college.[9] It will bear repeating, for emphasis, to say that our curriculum has been designed largely for the smaller group. In the event we have the predicted conditions for 1960, the first group will be even larger in comparison. It might be well to say at this time that the suggestions which are to follow should be considered even though an enormous increase in enrollment does not take place. The author is using the predictions in order to emphasize the need and excite activity. However, if the enrollment does increase, everyone should realize that which we must do.

The existence of these two distinct classes of students lets us know at once that at least two types of programs must be offered. Analyzing further we see that this is not enough. Each of our groups may be divided into sub-groups. Considering Group 1, first we have the following divisions.

(a) Those going into vocational work, farming and other kinds of manual occupations.
(b) Those going into office work, salesmanship, marketing and similar pursuits.
(c) Those who desire more mathematics just because they like it.
(d) Those who desire two more years of a liberal education.

With the exception of one course, very little, if any, will be said about subject matter content in this paper. The litera-

[9] The predicted junior college of 1960 will contain a third group; however, their mathematical needs will be satisfied on the basis of the two groups as listed. This third class is made up of older people, persons who have already entered upon their life's work, but are desirous of refresher courses or new information.

ture is full of course outlines for the students of sub-groups (a) and (b). In fact good texts are in print and are already in use in some schools. The important thing is to make the two types of courses available to the students concerned. The students of sub-group (c) will be interested in taking the traditional courses of algebra, trigonometry, analytics and the calculus. A consideration of sub-group (d) will be taken up later.

Group 2 may now be divided as follows:

(e) Those intending to become scientists or study engineering.
(f) Those who wish to major in mathematics.
(g) Those going into commerce and business administration.
(h) Those majoring in fields wherein a thorough course in statistics is essential.
(i) Those majoring in the fields of liberal arts.
(j) Those intending to teach subjects other than mathematics in the elementary and high schools.

Here again, with one exception, the content of the necessary courses is well defined. The students of sub-groups (e), (f) and (h) would have to pursue the so-called traditional courses as listed above. Those of sub-group (g) would desire the usual course in stocks, bonds, annuities and the like.

Sub-groups (d), (i) and (j) will now be considered together. For a long time there have been a few mathematicians who have felt the need, importance and desirability of a year's course (something like six semester hours) for students of the humanities, of other fields not requiring specialized subject matter, and those planning to teach subjects other than mathematics in the elementary and high schools. Most mathematicians have always frowned upon this proposed course for two reasons. They are: (1) the expression "cultural course" seems to get into the picture, and (2) they were afraid it would become a so-called "crip" or easy course. Due to circumstances involving these ideas, they were not to be blamed. Nevertheless, after several good articles on this subject,[10] and the introduction of the course into a few of the universities, it is becoming more and more accepted.

This course goes under various titles. Also, the subject matter is not standardized, which is perhaps a good thing since its effectiveness depends upon the personality and ideas of those doing the teaching. However, the subject matter is designed to do three things in general: (1) give the student a working knowledge of the more essential topics found in the usual freshman course, plus a good review of the elements of arithmetic (primarily for the benefit of the prospective elementary teacher, although it is good for the others as well); (2) the history of mathematics stressing its relationship to the progress of man; and (3) the importance of mathematics in our lives today, pointing out the amount and type of mathematical training the various occupations and professions demand upon those who wish to enter them. We have had a course of this nature in effect at Louisiana State University for five years now, and we have found the results most gratifying. In the group of students who take it are to be found those who hated mathematics, found it cold and uninteresting, and saw no use for it. This is the type of student who, upon becoming a secondary teacher, hurt the cause of mathematics at this level through improper guidance. We feel that the course will do more to aid mathematics in the secondary schools than anything else which has ever been tried.

[10] Some of them are: Ore, Oystein, "Mathematics for Students of the Humanities"; *American Mathematical Monthly*, Vol. 51, No. 8, October, 1944.

Allendoerfer, C. B., "Mathematics for Liberal Arts Students"; *American Mathematical Monthly*, Vol. 54, No. 10, December, 1947.

Newsom, C. V., "A Course in College Mathematics for a Program of General Education"; THE MATHEMATICS TEACHER, Vol. XLII, No. 1, January, 1949.

Section II. The Procurement and Training of Teachers

You will say, and rightly so, that this curriculum will be impossible without a much larger staff of teachers. You will say further that teachers and the money with which to pay them are extremely scarce. This was true up until the past year and is still true to some extent. However, there are signs of relief. The matter of money is a public one, and the public is beginning to respond to our needs in a fine way. We have seen enough to suggest that with a continued educational approach and justifiable reasons for additional funds, the necessary money should be forthcoming. Increased salaries, of course, will be a distinct aid in the procurement of teachers. It is already making things easier. Nevertheless, there is still much that the teachers of mathematics can do to procure teachers. Even in high school it is not a bit too early to suggest to the mathematically-minded student the importance, the desirability and the advantages of a life as a teacher of mathematics. All teachers of mathematics have been untrue to their profession, whenever they made no attempt to encourage their best students to remain in the field. Should all of us work in this direction, a change for the better will no doubt take place. Unless the mathematicians become more active in this respect, 1960 will find us conducting classes in junior college mathematics, using instructors trained in most everything from physical education to English and not having had enough college training in the field to be even a good high school teacher.

Once the prospective teacher is secured, we are confronted with his training. There is one point that should be made clear in the beginning: that is, we do not need teachers in the junior college trained as pure or research mathematicians. If we did, we would be doomed to failure. In the first place there would not be enough of that type of mind to go around; and in the second place a student so trained would not make a good teacher, for he would not be happy in teaching courses no more advanced than the calculus. The mathematician is conceding one point, that is, he is beginning to recognize the need and desirability for a general course as discussed above. He needs to concede another one; namely, that there should be two groups of students working toward advanced degrees in his department: (1) the research student, and (2) the prospective teacher of mathematics through the junior college level. The writer is happy to report that this is beginning to be done in some universities. When it is done generally another big step forward will have been made in the interest of secondary mathematics.

The next problem, of course, concerns the curriculum for the prospective teacher. This topic will not be dwelt upon in this paper, because several studies have been made in which specific programs have been suggested. Much good information may be found in the *Fifteenth Yearbook*.[11] The writer had a paper published a few years ago which also gives specific suggestions for the training of Junior college teachers.[12] As we look to the future, the problem of teacher training should be given careful consideration.

Again, in closing, if certain predictions are to become fulfilled, we are facing a big problem. Ten years in advance is none too early to begin working on these matters. And, as said above, if the predictions do not materialize, a problem of a smaller nature still exists so that any energy expended is no more than should be done.

[11] "The Place of Mathematics in Secondary Education," *op. cit.*, Chapter X.

[12] Karnes, H. T., "Preparation of Teachers of Secondary Mathematics," The Mathematics Teacher, Vol. XXXVIII, January, 1945.

Know Your National Council Representatives

By Kenneth E. Brown

Chairman of State Representatives, Mathematics Department, University of Tennessee, Knoxville, Tennessee

Indiana Representative

Mr. Philip Peak is Head of the Mathematics Department in the University High School, Indiana University, Bloomington, Indiana. Mr. Peak is a graduate of Iowa State Teachers College and Iowa University. For the past two years he has been the Indiana Representative for the National Council of Teachers of Mathematics. During this time the mathematics teachers in Indiana have organized a state organization, affiliated with the National Council of Teachers of Mathematics. (If your state or district has not affiliated with the National Council of Teachers of Mathematics, write to Mr. H. W. Charlesworth, East High School, Denver, Colorado for the affiliation information.)

Philip Peak

Ontario Representative

Robert E. K. Rourke

Mr. Robert E. K. Rourke, Head Master, Pickering College, Newmarket, Ontario, has a great interest in mathematics education. This is evidenced by his activity in the National Council of Teachers of Mathematics, the Council of the Canadian Mathematics Congress, and the writing of three textbooks and a number of articles in professional periodicals. For several years he has advocated an "International" Council of Teachers of Mathematics, rather than a "National" Council of Teachers of Mathematics. Perhaps the recent affiliation of the Mathematics and Physics Section of the Ontario Education Association with the National Council of Teachers of Mathematics is a step in this direction. For several years the Teachers of Mathematics in Ontario have secured leaders in mathematics education from the United States to participate in their annual meetings. Recent guest speakers at these meetings have been Dr. Breslich, Dr. Betz, Dr. Fawcett, Dr. Beatley and Dr. Lazar.

LOUISIANA REPRESENTATIVE

JESSIE MAY HOAG

Miss Jessie May Hoag, Southwestern Louisiana Institute, Lafayette, Louisiana, has been the Louisiana Representative of the National Council of Teachers of Mathematics for nearly ten years. During this time the membership of the Council in Louisiana has increased four hundred per cent. Miss Hoag says this is due to the enthusiastic support and cooperation of teachers in "key" positions. Of course Miss Hoag's twenty years' experience in teaching in the schools of Louisiana has made it possible for her to know the energetic teachers in the "key" positions. Her experience as a student in Oberlin College, University of Alabama, Columbia University and University of Chicago has given her an insight into better ways of teaching mathematics.

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Contests and Scholarships

By J. R. Mayor
University of Wisconsin, Madison 6, Wisconsin

This month's report on contests includes a test sponsored for its own seniors by the high school's department of mathematics; a test for high school seniors from a number of schools, sponsored by a college; and a test for college freshmen sponsored by the college's honorary mathematics fraternity. These brief accounts describe one means that these schools have found to add to interest in the study of mathematics and to provide deserved recognition for achievement in this area. These three contests are fairly representative of contests in mathematics held throughout the country.

Following the brief account of the contests, reference is made to the attitude of The National Association of Secondary-School Principals toward contests and competitive scholarships.

Steinmetz High School, Chicago

The Department of Mathematics instituted a "Mathematics Award" last June. The award is a gold medal and will be granted each semester to the graduating senior who scores highest on a competitive examination administered by the faculty of the Mathematics Department. From time to time the name of the winner will be inscribed on a suitable plaque which reposes in the school trophy case. So far the contest is limited to our own high school.

To quality for the examination a student must have maintained an "E" average in four years of high school mathematics: 2 years of algebra, 1 year of plane geometry, and ½ year each of trigonometry and solid geometry. The first test given was a multiple-choice test of about 30 items which were intended to test understanding rather than mechanics.

The contest creates considerable interest and enthusiasm, and causes the contestant to make a comprehensive review of his mathematics courses to date.—Reported by Florence B. Young.

Mundelein College, Chicago

To encourage interest in Mathematics, the Department of Mathematics at Mundelein College has been sponsoring tournaments for the 36 Catholic High Schools in the Chicago area having students in attendance at Mundelein. These contests have followed the pattern used by Wilson Junior College for high schools in its vicinity. Competition includes eight rounds of three problems each from the areas of Arithmetic, Algebra, and Geometry. To be eligible, students should have completed five semesters of high school mathematics.

For review purposes, one year, Mundelein freshmen in mathematics courses compiled and sent out original problems to all the high schools. The members of the mathematics Faculty were responsible for constructing the problems actually used in the contest. In subsequent years alumnae of the Department of Mathematics, who are now teaching in high schools, returned to the college and designed the contest problems, thus providing a source of inspiration and unity among all interested in mathematics.

Awards are presented to the leading schools, and also to individual students who are winners.

Visiting Faculty members and contestants are guests at a tea following the tournament, with the members of the Mathematics Department acting as hostesses.

An item analysis of the problems used in the tournament is made, and results are sent to the participating schools. (This work is done by senior Mathematics students who are engaged in student teaching.)

A report of this project, including a comparative study of the deficiencies as evidenced in our tournaments with similar studies throughout the country, will be made at the Illinois Academy of Science meeting in the Spring.—Reported by Department of Mathematics, Mundelein College.

Hofstra College, Hempstead, New York

The New York Alpha Chapter of Kappa Mu Epsilon each year gives an award to the outstanding student of freshman mathematics. This student is chosen on the basis of his grades in his freshman courses in mathematics and a two hour competitive examination. The freshman course includes four credits of Advanced Algebra and Trigonometry and four credits of Analytic Geometry. The examination stresses problems of thought and originality.

The prize, usually a mathematics book, and an engraved award certificate are presented at the "Spring Day" exercises held during Senior Week.—Reported by E. Marie Hove.

Contests as Viewed by the National Association of Secondary-School Principals

By writing to the National Association of Secondary-School Principals, 1201 Sixteenth Street, N.W. Washington 6, D.C. one can obtain a copy of their pamphlet "National Contests for Schools, 1949–50." This pamphlet prepared by the Association's National Contest Committee of which Mr. G. A. Manning, principal of the Muskegon, Michigan Senior High School is chairman, lists recommendations for participating in national contests, criteria used in evaluating all national contests for inclusion in The Approved List of National Contests for Secondary Schools, and the approved national contests for 1949–50. Reports from the Committee also appear twice a year, in the October and February issues of *The Bulletin* published by the Association.

Teachers whose students enter or plan to enter national contests, will be interested in reading the recommendations of this administrators group.

More important, any groups sponsoring contests, even though on a local rather than national scale, should make themselves familiar with the point of view of administrators toward such competitive activities.

On the approved list are a French Examination sponsored by the American Association of Teachers of French and a Latin Examination sponsored by the National Association for Promotion of the Study of Latin. Also included are the Science Talent Search and the Scholastic Magazine, Inc. contests in art, literature, and music. In addition to a number of competitive scholarships there are listed some 25 other national agriculture, art, essay, and forensic contests.

Louisiana State University Mathematics Institute

Louisiana State University will hold its first annual Mathematics Institute from June 25 to July 1, 1950. There will be discussion groups, lectures and exhibits for both the elementary and secondary school teacher. Programs will be available about May 1. If interested, please write Houston T. Karnes, Department of Mathematics, Louisiana State University, Baton Rouge 3, Louisiana.

Have You Heard?

By EDITH WOOLSEY
Sanford Junior High School, Minneapolis, Minnesota

DISCUSSION groups are becoming more popular and more valuable with each meeting of the Council. It is interesting to note that teachers in distant parts of the country have the same teaching problems to solve. Here are brief summaries of a few of the discussions at the December program in Wichita. If you take up these questions at meetings of your local clubs, and reach conclusions which would be of value to other teachers, send them in and let us relay them to other readers of THE MATHEMATICS TEACHER.

I. The Advantages and Disadvantages of General Mathematics at the College Level.

Leader: Prof. James H. Zant, Oklahoma A. and M. College, Stillwater, Oklahoma.

There were 25 participants in this discussion representing a number of types of colleges, large state universities, private colleges, teachers colleges and other state colleges. The questions raised were from two viewpoints: (1) General mathematics for the non-scientific or general student, and (2) General mathematics for the students who expect the major in mathematics, science, engineering or some other field that requires at least two years of college mathematics.

Under (1), some of the questions raised were:

1. What is the purpose of a general mathematics course for such students?
2. What is the content or subject matter?
3. What has been done in other colleges?
4. How is the course administered?
5. Are there any prerequisites?
6. Shall it be a course *in* mathematics or *about* mathematics?
7. Can credit be transferred to other colleges?
8. What could be done about a text book?
9. How shall we evaluate the results of the course?

The second viewpoint was not discussed due to lack of time. It was not possible in the time available to make recommendations on, or even discuss, all of the questions raised. The group was in fair agreement that such a course should cover a wide range of mathematics, including something of the nature of mathematics, logical thinking and the history and development of the subject. However, the group did agree that any course of this sort should be a mathematics course and not a course about mathematics.

II. Can Pupil Failures in Plane Geometry Be Reduced by Use of the Laboratory Method?

Leader: Allene Archer, Thos. Jefferson High School, Richmond, Va.

The laboratory method was defined as the discovery of truths by students doing with their hands some experiments that lead to geometric concepts. Different propositions that can be proved by models were discussed. These included the congruence propositions, the base angles of an isosceles triangle, parallel lines problems and others. Models were displayed and their uses explained. Methods of presentation of propositions were discussed. Since only four people in the group had used the laboratory method, the question of whether its use would reduce failures could not be decided by the group. Those who had used this method have found that it helped the pupils become more interested in geometry and, because of their

interest, they worked harder and fewer failed.

III. The Future of Mathematics Contests.

Leader: Adrien Hess.

Mr. Hess gave a survey of the background on the development of contests, mentioned many of the criticisms of contests in general, and issued a challenge to mathematics teachers to do something to relieve the shortage of scientists. This topic was of particular interest because the National Council is considering the question of whether or not to have a mathematics contest. The participants felt that a national contest would be a help in locating mathematical talent. There seems to be a leveling-off process in our schools instead of an encouragement and pushing forward of the best pupils. The value of the inspiration and encouragement of a good teacher cannot be overestimated in considering the development of gifted pupils.

IV. How Can Teachers Stimulate Pupils to Think Carefully and to Enjoy Geometry?

Leader: Grace McMahon, Lincoln High School, Lincoln, Neb.

The following ideas were expressed by various people in the group:

1. Reasoning rather than memorizing is stressed in proofs.
2. Many originals should be given.
3. There are advantages in group work, two or three assisting each other at the blackboard or at their desks.
4. Models made by pupils and commercial models stimulate interest.
5. Originals were suggested to care for the problem of individual differences in class.
6. Some make assignments a week in advance in order that students can make preparation when it best fits into their time schedules.
7. Some grade daily work in class, others check daily papers but do not grade daily work, and others correct daily work and hand it back ungraded.
8. Much of the teaching is done in the class period. It is becoming increasingly difficult to get home work done.
9. Some complete proofs should be used in tests rather than having tests that are entirely objective.
10. Some device that challenges the attention should be used when new work is presented.
11. Attempt to develop clear thinking.
12. Proofs in texts may be used as forms and to teach methods of thought.

V. Which Visual Aid Will Be Used Today?

Leader: Nelle Kitchens, Hickman High High School, Columbia, Missouri

Visual aids are "springboards for discussion," not a substitute for serious mental effort. They should be introduced early in the course before interest is lost. They reduce tension and increase interest. Specific aids mentioned are: blackboard, Bulletin board, slides, models, supplementary texts, field trips, library, films, anecdotes and quotations, files containing current articles, flash cards, mathematics club programs, resource units, and exhibits.

VI. Teaching and Examination Techniques in First Year College Mathematics.

Leader: Dr. Guy W. Smith, Washburn Municipal University, Topeka, Kansas.

There was considerable discussion of the first year courses: algebra, trigonometry, and analytical geometry, but little discussion of such courses as general mathematics, statistics, and the theory of investments. A great deal of difference as to prerequisites, arrangement of subject matter, amount of credit, etc. among the various schools was indicated. In some cases, several different courses were given by the same school. Concern for the poorly prepared student was expressed. The question

of non-credit courses came up frequently.

Reports on guidance of instructors were given by representatives of several of the larger schools. All made use of one or more of the following devices: course outlines, group meetings, coordinators, class visitations, double sections with an assistant.

The technique of conducting classes was discussed at some length, with the following topics being considered: textbook selection, method of presenting new material, policies concerning daily assignments, black-board work, tests and quizzes. Most of the larger schools had textbook committees. The majority of the teachers generally made some explanation of new material before it was assigned, but did not disagree when the opinion was expressed that this should not always be done, that the student should learn to read for himself. Most of the teachers indicated that they had problems handed in most of the time, and several others indicated that they did so more than half of the time. Almost all indicated that they made use of blackboard work and considered it very valuable, but there were a few dissenters.

Final examinations were discussed. Nearly everyone seemed to favor uniform final examinations, but in only a few cases was it possible to arrange schedules so that it could be done. Most instructors used mimeographed examinations. In general, the examination seemed to count about one-third of the course grade, but in a few cases it was 50%. Multiple-choice and traditional examinations were mentioned, but little indication was given as to the types being used. In connection with the question of dishonesty, it was agreed that it was desirable to make cheating difficult by separating seats, careful but not too obvious proctoring, and by using multiple forms of examinations.

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Group Affiliation with the National Council

By H. W. Charlesworth

Chairman of Affiliated Groups, East High School, Denver, Colorado

By the time this article appears, the Chicago convention will be over and the *first* meeting of the Delegate Assembly will have put added vigor into affiliation matters. The effectiveness of the work of this assembly will depend upon how well their decisions are carried out cooperatively by the affiliated groups and the National Council. We hope that the delegates will make known to their respective groups as soon as possible, what transpired at this delegate assembly meeting. We trust that the accomplishments of this assembly will prove to be definitely helpful and inspirational to all groups.

Last September a questionnaire on affiliation was sent out to groups, to state representatives, and to members of the Board. At the time of writing, about half of these have been returned, a very good response considering the type of questionnaire and the fact that a stamped self-addressed envelope was not provided. A complete summary of the reactions on the questionnaire was sent to affiliated groups in time for them to study it before the Chicago meeting. The results of this questionnaire served as a basis for the work of the Delegate Assembly. Since only the affiliated groups received this summary, I am reporting on significant parts of it now, in order to give information about it to all members of the National Council and with the hope that it will be of help to groups not yet affiliated.

Concerning eligibility of groups for affiliation, the results of the questionnaire indicate that the membership of the group should be broad, that it include teachers of mathematics from elementary through college and any others who are interested in promoting the purposes of the group, but making sure that the majority of the members are mathematics teachers. Most constitutions of affiliated groups do allow for this breadth of membership. The minimum number in the group should be fifty, at least half of the members and all of the officers being members of the National Council. The group should have a written constitution and by-laws, maintain a list of members with teaching positions and addresses and indicating those who are members of the National Council. The group should have one or two meetings a year, some may want more. It appears highly desirable that the group coordinate some of its activities with other groups and have a coordinating committee for the purpose. Groups should be privileged to report their activities for publication in The Mathematics Teacher and in a newsletter that would go out to groups frequently. Each group should publish its own bulletin or newsletter two or three times a year, this to be sent out to each member of the group and exchanged with other groups. The group should be so organized as to provide for continuance of leadership and avoid the necessity of electing a complete new set of officers every year or two.

The results of the questionnaire indicate that it is desirable for affiliated groups to pay dues to the National Council, the amount of which probably depending upon the size of the group. This, of course, with the understanding that the National Council will use the monies from such dues to help carry out a good program of affiliation. It is indicated that the group should be expected to "talk up" the National Council and solicit members to it, that the group display and take orders for the Yearbooks and other publications of the National Council. More than 70% think that the work of the National Council and its affiliated groups should be more effective if the National Council were a department of NEA. Many groups are hop-

ing that it will be possible to arrange to have a representative of the National Council, a designated person or persons, to visit with groups to help in various ways such as, organizational work, coordination of work among groups and with the National Council, arranging regional programs, stimulating the interest of the group in certain activities.

Concerning the activities of groups, the questionnaire reveals that most groups would welcome the opportunity of taking part in surveys and studies conducted by the National Council. Groups have engaged in various activities in the past. In addition to regular meetings, most groups have committees at work during the interim of meetings. They feel that their most effective work is done through these committees. Committees work on such things as, guidance programs in mathematics, coordination of high school and college mathematics, the general mathematics program, recommendations to state department of education, studies of grade placement, etc. Other activities are: sponsoring and directing institutes and workshops, publishing a bulletin or newsletter, putting on mathematics exhibits, cooperating with state department on certification requirements, assisting in testing programs of students entering college, sponsoring awards and scholarships, helping in the selection of textbooks for state adoption. Most of the groups are doing more than just arranging and conducting meetings of the group. Some indicate that they are working on closer cooperation of groups within their own state or region, some say that they expect to set up certain standing committees, such as, curriculum, research, coordination, editorial, membership. Many groups indicate that they would like to cooperate with other groups on such matters as, improvement of teaching experiments, curriculum improvements, in-service training, exchange of speakers, larger area meetings, grade placement studies. Several groups do have written reports of their studies that can be made available to other groups. The great majority would welcome the opportunity of taking part in surveys and studies that the National Council might conduct. Many fine suggestions were made on the kinds of studies and surveys that could be done. Also, many good suggestions were made as to the services that the National Council might render to groups, the one most frequent being that the National Council should give help in getting capable speakers for group programs.

The foregoing contains some of the suggested conditions of improved affiliation. Just what the actual conditions will be depends upon the Delegate Assembly and the Board of Directors. We shall make reports on this from time to time.

Indications are that we shall soon have *state* organizations of mathematics teachers in most of the states. Leaders in the states where such state organizations do not now exist may want to help get such a group started. These people will find some help in this series of articles which began in the October issue. If further help is desired, please write to me. If I cannot help you directly, I am in a position to suggest individuals or groups who can. Very soon now we should have a pamphlet on affiliation which would tell what affiliation with the National Council means, how to organize new groups, how to affiliate groups, possible activities of groups, and any other pertinent information about group affiliation.

Miscellanea—Mathematical, Historical, Pedagogical

By PHILLIP S. JONES
University of Michigan, Ann Arbor, Michigan

THE PYTHAGOREAN THEOREM

A RECENT dispatch by Albion Ross from Baghdad, Iraq notes the discovery there two months ago "that schoolboys of the little county seat of Shadippur about 2000 B.C. had a textbook with the solution of Euclid's classic triangle problem seventeen centuries before Euclid." It is further stated that "The chief cuneiform expert of the Iraqui Museum, Arche Dogy, says that the presentation of the famous problem is tinged with algebraic concepts that appeared even later than Euclid in the development of mathematical science."[1]

Assuming that "Euclid's classic triangle problem" refers to the Pythagorean theorem, this raises again the question of what is meant by the Pythagorean theorem and what is its ultimate source. In years past a knowledge of the 3-4-5 right triangle was attributed to the Egyptian rope stretchers or harpedonaptae. This tradition is now recognized as completely unverified and perhaps even improbable.[2] Further, the determination of right angles by such a triple of lengths, did it exist, could have arisen out of purely empirical considerations requiring no thought of "squares." There are cogent arguments tending to show that many Egyptian generalizations, such as their value of pi, were based on experimentation and averaging.

The Pythagorean theorem and its converse as we think of them are generalizations for all right triangles established by a logical proof. Special cases and related concepts which are, however, not completely equivalent to it include: (1) "Pythagorean numbers" or triples of integers which satisfy $x^2+y^2=z^2$, (2) the use of the ideas of the theorem in numerical situations without a stated generalization, (3) a general statement of the theorem without a proof, (4) the computation of approximations to the irrational numbers which would arise if the theorem were understood and used in a really general fashion.

The problem of finding a procedure or formula which would yield all sets of Pythagorean numbers is more properly a part of number theory than of geometry. A solution of this problem appeared in the lemma following Euclid X, 28. Another formula is attributed to Plato and it is probable that the Pythagoreans had a solution for it. In this connection one of the most outstanding discoveries in recent mathematical historical research was published by Neugebauer and Sachs[3] in 1945. This is a cuneiform tablet of Pythagorean numbers arranged in such a fashion as to indicate conclusively that the table was systematically constructed by persons in possession of a "formula" for these numbers. (Of course neither Greeks nor Babylonians had formulas as we visualize them involving letters and symbols.) This tablet dates back to the period of 1900 B.C. to 1600 B.C. and as such is now the first recorded example of work in number theory. The underlying formula is $a^2+b^2=c^2$ where $a=2pq$, $b=p^2-q^2$, and $c=p^2+q^2$. p and q may be chosen at will, and, subject to the conditions that p and q are relatively prime, not both odd, and $p>q$, these formulas will then give all sets of relatively prime Pythagorean numbers.

The Babylonian computation of $\sqrt{2}$ was discussed earlier in this Department.[4] They also used $a+b^2/2a$ as an approxima-

[1] *The New York Times*. Jan. 8, 1950. Section 1, pp. 1, 28.

[2] Edmond R. Kiely, *Surveying Instruments. Their History and Classroom Use*. New York: Bureau of Publications, Teachers College, Columbia University, 1947. p. 13.

[3] O. Neugebauer and A. Sachs, *Mathematical Cuneiform Texts*. New Haven: American Oriental Society, 1945. pp. 38–40.

[4] *The Mathematics Teacher*. 42 (Oct., 1949), pp. 307–308.

tion to $\sqrt{a^2+b^2}$. This approximation lacks only decimal place notation and the idea of repeating the approximation procedure to be our modern algorithm for square root. (This latter actually originated with the Hindus from whom we got our decimal place system.)

Among the Babylonians these computations grew out of problems requiring the finding of the diagonal of a square, the distance of the foot of a rod of known length from the base of a wall against which it leans, the length of the chord of a circle, given its radius and the distance to the chord.

These calculating procedures and problems indicate that the Babylonians had a general concept of the Pythagorean theorem and even applied it whether the result was rational or irrational. There is no indication, however, that they had any hint at all of the distinction between rational and irrational numbers, nor is any generalized or abstract statement of the theorem known.

Mr. Ross' article in the *Times* further states that the new material "suggests that mathematics reached a stage of development about 2000 B.C. that archaeologists and historians had never imagined possible." The previously known facts which are cited above plus the well known interest of the Babylonians in problems that were essentially algebraic would indicate that this new discovery, though interesting, may not be quite so unheard of nor unthought of as the newspaper writer chose to portray it. However, *if*, as the newspaper may imply, there is a diagram or statement of a general sort as opposed to a mere numerical problem, and, even more, if there is any attempt at *proof*, then this may indeed be of extraordinary historical significance. As far as is yet known these ideas of a stated theorem or general rule and of proof still have their beginning no earlier than the time of the ancient Greeks. Their occurrence in ancient Sumerian times would be as great a surprise as the newspaper quotation implies. That exchanges of ideas between Babylonian, Egyptian, Greek, and Hindu civilizations existed is being more and more thoroughly established, but the existence of *proof* in geometry[5] prior to and outside of Greece is still unknown.

The Hindus *did* use the Pythagorean numbers and a rope to lay out right angles. The Pythagorean set 15, 36, 39 which can be derived from the relatively prime set 5, 12, 13 was used for this purpose in the Sulvasutras in which is also found a general statement of the Pythagorean theorem. The Sulvasutras however deal, not with surveying, but with religious matters, especially the construction of altars. They are variously dated from 800 B.C. to 500 B.C.[6]

Notes, Additions, Corrections

A card from R. C. Yates of the United States Military Academy states that the variable parallelogram mechanism which was described in this column in the December, 1949, The Mathematics Teacher (vol. 42, p. 402) was a part of a very interesting display arranged by George R. Anderson of State Teachers College, Millersville, Pennsylvania.

Mr. Anderson adds that at his suggestion one of his students, George Frenett, actually obtained the mechanism in a junk yard and mounted it for display. Mr. Anderson also notes that he has himself demonstrated mathematical devices on several television programs, and that beginning January 17 he is to give a six weeks television course on the slide rule over WGAL-TV, Lancaster, Pennsylvania. This note will appear too late to advertise his course, but it does suggest the added possibilities for mathematical radio programs which

[5] R. C. Archibald, *Outline of the History of Mathematics*. Mathematical Association of America, 1949, pp. 13, 14, 59, attributes the "foundations of algebra" to Babylonian mathematics which he says "displays substantial illustrations of general techniques for proving its procedures."

[6] J. L. Coolidge, *A History of Geometrical Methods*. Oxford: 1940, pp. 13, 14.

are brought in by TV. Are these the first appearances of mathematics on television? Mr. Anderson promises to write a full scale article on his television experiences at their conclusion.

Donovan Johnson of the University of Minnesota sent in a copy of the *Minnesota Math-News Letter* Vol. I, No. 3 along with several other helpful hints. This five page mimeographed publication of the Minnesota Council of Teachers of Mathematics carried news notes, comments on the task of their new state curriculum revision committee, a list of state and national meetings, detailed instructions by Emil J. Berger of Monroe High School, St. Paul, Minnesota for making several excellent teaching aids, and a short list of "New Materials for the Mathematics Classroom." Even non-Minnesotans might find these suggestions and the data on mathematics activity in a sister state worth a dollar. This is the price of a year's subscription (membership) which could be sent to Miss Edna Lingren, 1922 Sargent Avenue, St. Paul, Minnesota.

The writer's proof-reading eye slipped over an error in this column in the December, 1949 issue, page 401. The element in the second row and first column of the determinant there displayed should have been $-ln^{-1}l$ not $-lu^{-1}1$, and the element in the second row second column should have been L^2. The value of the limit of the determinant would then have been $LL+ey$. To capitalize on this slip, however, we might remark that the abbreviation $ln\ x$ for the natural logarithm of x is an American invention appearing first in Stringham's *Uniplanar Algebra* published in San Francisco in 1893. Many other notations had been suggested and used earlier, the first being merely lx suggested by Leibniz. "-1" written as an exponent to represent an inverse function was first used in 1813 by John Herschel, the astronomer, for inverse trigonometric functions.[7] The use of this symbol in matrix theory and modern algebra is now quite common. The writer has not seen $ln^{-1}x$ used for antilogarithms before though. Have you?

The writer's attention has also been called to the fact that the harmonic mean of a and b should have appeared as $2ab/(a+b)$ on page 65 of THE MATHEMATICS TEACHER for February 1950 (vol. XLIII), and that the number of pieces of candy mentioned there should have been $2\frac{2}{5}$.

[7] Florian Cajori, *A History of Mathematical Notations*. Open Court, 1928, II, pp. 107, 176.

Indiana University Mathematics Workshop

Indiana University will hold its third annual Workshop for Mathematics Teachers June 19 to July 1. Graduate credit will be given for those who desire it. There will be exhibits, lectures, group discussions and analyses of problems brought in by the members. There will be a shop and instructor available for those who wish to try their hand in making visual aids. Programs will be available about May 1. If you wish a program, please write Philip Peak, University School, Bloomington, Ind.

Arithmetic Section

Conducted by BEN A. SUELTZ
State Teachers College, Cortland, N. Y.

In this issue:

Three new courses of study in arithmetic are reviewed. These represent California, New York, and Pennsylvania. They indicate that real progress is being made in the teaching of arithmetic. This is particularly true on the curriculum level. However, the most important task is that of bringing our best thinking into the classrooms. How can this be done? It is interesting to note that each of the courses of study was prepared by a comparatively few people but in each case this was done after wide consultation with teachers and supervisors. Many curriculum workers believe that the wider the participation in the preparation of syllabi and courses of study the more real value to the teachers and to the boys and girls. Do we need many special workshops and institutes in order that our teachers in service might gain the benefit of our best thinking about aims, methods, procedures, materials, and modes of evaluation in arithmetic?

For this issue Professor Van Engen has prepared a bibliography on the teaching of arithmetic. Note the items, here is the basis for re-orienting thinking about arithmetic. Mathematics teachers should call this bibliography to the attention of other teachers, particularly those who are studying or teaching in the general field of elementary education.

In the May issue:

Several papers dealing with special phases of arithmetic will be printed. Have readers special requests for articles to be printed next year?

"California's Guide to Teaching Arithmetic in Kindergarten and Grades One and Two." Reviewed by Ben A. Sueltz.

This is a guidebook which was specifically prepared for teachers in the California schools. Mrs. Leslie Beatty, with the cooperation of many teachers and supervisors, prepared the booklet while on leave from her regular position at San Diego State College. The scope of the guidebook is evident from the chapter headings:

I. Arithmetic in the Life of the Young Child
II. How to Develop Understanding in Arithmetic
III. Evaluating Understanding in Arithmetic
IV. Materials of Instruction

The main aim throughout the guide is the development of understandings that shall be meaningful to children. The role of experience with real things in real situations is emphasized. However, the use of symbols and facts is not neglected; rather, the development of symbolic arithmetic from experience and its use in understanding the quantitative relationships of experience is capitalized. Such children's activities as games, distributing supplies, carrying on activities in science, telling stories, and solving problems (in the broad sense) provide a setting for and an impetus to learning arithmetic. In brief the program recognizes that real learning is individual for each child and that experience should precede symbolic work and give meaning and significance thereto.

Careful attention is given to the concepts and language of common measures. It is suggested that many common measures be readily available for children to see and use. Likewise an understanding of numbers and of number relationships and combinations grows from manipulation and experience with real things and with conventional blocks and pictorial materials. It is doubtful if teachers will find the suggested use of the abacus as worthwhile in developing understanding of numbers

and the basic principles of the number system as is hoped. However, the abacus is only one of many suggested implements and devices. A most valuable item for teachers is the chart: "Summary of Objectives and Typical Experiences" which shows the types of experiences found in the kindergarten and in grades one and two. Too frequently, teachers fail to recognize the essential arithmetic in a common experience and to note the need for guidance and instruction of pupils so that real learning is accomplished.

The California Guide points out that evaluation is closely related to teaching and learning. Emphasis is placed upon informal evaluation through careful observation of and discussion with pupils. But this observation, while it may appear to be casual, is nevertheless carefully planned and recorded.

In general, these California teachers have a course of study that shows teachers how to make arithmetic meaningful and important to young children. It recognizes that growth in understanding does not come automatically but must be planned. The need for real experience in seeing and manipulating as well as in talking about things arithmetical as an important step preceding the stage of symbols and writing is clarified.

Correspondence relative to this handbook should be addressed to The California State Department of Education, Sacramento, California. Refer to Bulletin: Vol. XVIII, No. 8, November, 1949. Guide to Teaching of Arithmetic in Kindergarten, and Grades One and Two.

New York's "Mathematics for Boys and Girls," by H. George Murphy, State Education Department, Albany, New York.

"Mathematics for Boys and Girls," a handbook for teachers of the Kindergarten and Grades I through VI, will be issued by the New York State Education Department this spring. This publication will be a revision of the now well-known New York State syllabus for elementary school mathematics. The same basic organization and philosophy that were presented in the old will be carried forward in the new edition. Certain sections will be entirely rewritten and the philosophy of *meaningful* mathematics teaching will be more carefully emphasized. Materials for Grades Seven and Eight will no longer be included, since a new Junior High School syllabus also is being prepared.

An advisory committee, composed of curriculum experts, mathematics specialists, superintendents of schools, elementary principals and supervisors, and classroom teachers, critically studied the old syllabus and made recommendations for its improvement. Groups of teachers in the field also made contributions. The actual rewriting was done by Dr. Ben A Sueltz, Professor of Mathematics, State Teachers College, Cortland, New York, with the assistance of Dr. Vera A. Sanford, Professor of Mathematics, State Teachers College, Oneonta, New York. Thus the new bulletin was written by two people who are specialists in elementary school mathematics and in teacher training. They were temporarily associated with the Bureau of Curriculum Development of the New York State Education Department while engaged in this work.

"Mathematics for Boys and Girls" will not be an outline of content to be taught. It will be a resource volume to be used by teachers to assist them in teaching boys and girls. The entire first section of the handbook—that part that was most completely rewritten, incidentally—is devoted to an explanation of the philosophy of *mathematics for meaning*, and of how children best learn mathematics. Many pages are also devoted to answering those questions that are frequently asked by teachers, such as: "How does a new teacher start her work?" "Should flash cards be used?" "Should my grade follow a syllabus?" "Are there preferred methods of subtraction?" "Should children use crutches?" and similar questions.

Probably, however, the most distinguishing feature of this forthcoming publication will be the almost unique organization of its second section. This section will be introduced with a large "Sequential Learnings Chart," which will be bound in the book. As its name implies, this chart outlines, in the probable order in which they will be learned by the children, those mathematics learnings that are characteristic of the life of children of elementary school age. These sequential learnings are outlined in nine inter-related categories, (1) Solving problems, (2) Numbers: counting, reading, and writing, (3) Addition and subtraction, (4) Multiplication and division, (5) Measurement, (6) Common fractions, (7) Decimal fractions, (8) Geometry, and (9) Algebra. It will be noted that "Solving Problems" is listed first. This is in accordance with the point of view basic to the entire handbook, *viz.* that modern mathematics teaching, to be based on *meanings* and *understandings*, is primarily a matter of helping children *find the answer*. The remainder of the second section of the handbook expands and develops the nine categories of sequential mathematics learning, as outlined on the "Sequential Learnings Chart." Suggestions are given for teaching children at their various stages of progress in terms of these sequential learnings. Many concrete examples are given.

Throughout the new handbook the child development approach will be stressed and the necessity of individualizing mathematics instruction will be pointed out. A "Suggested Grade Placement Chart" will be included in a pocket in the book cover for use by those teachers who need help in understanding what mathematics knowledge and abilities are held by children in general at each of the various grade levels. However, the necessity of teaching children where they are in their mathematical development and assisting them step by step will be emphasized.

"Mathematics for Boys and Girls," as are other course-of-study materials issued by the New York State Education Department, is intended for use by the teachers of that State and will be issued free of charge to the schools. It is expected, however, that copies will be available to the Departments of Education in other states and cities on an exchange basis. Individuals desiring the booklet will be required to pay a small charge to the Bureau of Publications of the New York State Education Department at Albany.

"The Arithmetic Section of Pennsylvania's 223—B," by Charles E. Poole, Principal Highland School, Abington, Pa.

The Pennsylvania state-wide curriculum improvement plan was launched in 1946 when Bulletin 233-A, "Local Participation In State-Wide Revision In the Elementary School Curriculum" was made available to the teachers, supervisors, and administrators of the schools of Pennsylvania. The 1949 Bulletin 233-B, "The Elementary Course of Study" grew out of such local participation. Specifically, the chapter on arithmetic was worked out by a committee composed of a first grade teacher, a fifth grade teacher, a district superintendent, and an elementary school principal. This was done under the leadership of Dr. Leversia Powers, Chief of Elementary Education. The committee also had the advice of a professor of mathematics from one of the state teachers colleges.

The basic point of view of the Committee as found on page 297 of Bulletin 233-B is as follows:

"We live in a social world which the science of numbers has largely helped to develop. For example, our trade and transportation, our recreation, and daily living from the alarm clock's bell in the morning all are dependent upon the science of numbers. We need to be able to use numbers to understand and interpret the world about us. Our ideas of space and form have a place in the scientific interpretation of our world, as well as in the appreciation

of the beauty expressed in nature, art, and industry. Forward strides which are being experienced in every field of endeavor from our daily living with problems of food, clothing, and shelter to television, airplanes, and atomic energy, show the need of a revitalized and enriched form of mathematics which requires a specialized kind of knowledge.

"Arithmetic is more than a set of specific skills and facts. It is a system of quantitative and qualitative thinking including facts, concepts, principles, and processes which are so closely allied that they cannot be separated in practice. This kind of thinking is a primary importance. It develops slowly and needs the incentive of being functional. (Arithmetic should be more of a challenge to a pupil's intelligence than to his memory. Frequent stimulation, guidance, and evaluation by an expert teacher are required."

The Arithmetic Committee recognized that the Course of Study would be "used" only in so far as it has a direct appeal to the thousands of elementary teachers in the state. The following distinguishing features make it appealing:

1. Suggestions to teachers for social usage.
2. Listing of vocabulary which is probably basic to the material described for a grade level and a recommendation for the teaching of this same vocabulary.
3. The proposed new method of treating the development of numbers one through ten and the suggested pattern for the development of the concept of a number. Obviously, those teachers of the state who have had recent courses in arithmetic methods could best understand and apply these suggestions.
4. Constant use of the phrase "test and then re-teach when necessary."
5. An informal evaluation of teaching techniques.
6. Emphasis on flexibility of the program to promote maximum educational development of every child.
7. The inclusion of sample units.

The State Department of Public Instruction in recommending this new course of study is encouraging administrators and supervisors to:

1. Set up a plan to have teachers study and discuss this arithmetic chapter in groups.
2. Provide adequate material for building concepts.
   a. Manipulative material
   b. Printed material
3. Provide opportunities for experiences:
   a. Excursions
   b. Audio-visual materials
   c. Units
4. Encourage grouping within groups.
5. Set up criteria for evaluation of texts and workbooks and of their proper use.
6. Set up criteria for evaluating pupil progress in the light of desirable objectives and philosophy.

Anyone from outside the State of Pennsylvania may purchase a copy of Bulletin 233-B for $2.50 per copy. Checks should be made payable to the Commonwealth of Pennsylvania and should be addressed to the Editor of the Department of Public Instruction.

A Selected List of References on Elementary School Arithmetic

By H. Van Engen
Iowa State Teachers College, Cedar Falls, Iowa

With summer school "just around the corner" it would seem advantageous to the elementary-school teacher to have a list of selected references on arithmetic at hand in order to facilitate the study of instructional problems. With this thought in mind a list of books published since 1946, and articles published since 1947, was prepared. Some excellent articles dealing with the frontiers of educational research in arithmetic were not included because the list was compiled to serve teachers interested in suggestions for the improvement of instruction in arithmetic.

BOOKS

1. Brueckner, Leo J. "Arithmetic Knowledge and Abilities of Children in Grades I and II." Educational Publishers, Inc. Minneapolis, 1948.

A study of the status and rate of growth of arithmetical knowledge and abilities of children in grades I and II. In addition to the children's ability to count and compute the author was interested in obtaining data relative to their knowledge of social applications of number and their ability to solve problems. With regard to the latter it is interesting to note that "It seems evident that the situation involved in the problem and the relations that are expressed in it are the decisive elements in determining whether or not a particular problem is difficult" (see page 93).

2. Brueckner, Leo J. and Grossnickle, Foster E. "How to Make Arithmetic Meaningful." The John C. Winston Company, Philadelphia, 1947.

This book discusses the methods, content, grade placement, learning theory, and the evaluation of learning, of the arithmetic usually taught in the first six grades of the elementary schools. As the title indicates the book's central theme is "meaningful instruction of arithmetic."

3. Buckingham, Burdette R. "Elementary Arithmetic: Its Meaning and Practice." Ginn and Company, New York, 1947.

A book written in recognition of the fact that "the meaning theory so urgently desired cannot be taught in the schools by teachers who have never learned arithmetic with meaning." This book is not a book on psychology or the teaching of arithmetic. It is an extended treatment of the arithmetic in the elementary schools. It should be on the desk of every teacher who claims to be a teacher of arithmetic.

4. Rosenquist, Lucy Lynde. "Young Children Learn to Use Arithmetic." Ginn and Company, New York, 1949.

A discussion of the arithmetic to be taught in the kindergarten, first grade, and second grade of the elementary schools, as well as the methods of instruction to be used in presenting the arithmetic advocated for the kindergarten and grades I and II. The bibliographies included at the end of each chapter should be helpful to anyone interested in a complete study of a particular problem in arithmetic in the above mentioned grades.

5. Spitzer, Herbert F. "The Teaching of Arithmetic." Houghton Mifflin Company, Boston, 1948.

This book, like all recent books on methods in arithmetic, emphasizes a meaningful approach to the arithmetic of the first six grades. The liberal use of objects to develop the meanings of symbols, to develop the problem solving ability, and to "prove" answers to number combinations and problems is the chief characteristic of the methods advocated by the author.

6. Stern, Catherine. "Children Discover Arithmetic: An Introduction to Structural Arithmetic." Harper & Brothers, New York, 1949.

A methods book on the teaching of arithmetic based on the principles of Gestalt Psychology. This book should be carefully examined by all teachers of arithmetic, especially nursery-school, kindergarten, and primary-grade teachers.

7. Taylor, E. H. and Mills, C. N. "Arithmetic For Teacher-Training Classes." Henry Holt and Company, New York, 1949.

This book emphasizes the importance of "(a) generalization that comes through understanding . . . (b) the value of 'teaching with meaning,' (c) the coordination of abstract number processes with social utility, and (d) seeing arithmetic as a unified system. . . ." The book is written to provide a mathematical and pedagogical background for the teacher in training, however experienced teachers will find much of value to them.

MAGAZINE ARTICLES AND BROCHURES

1. Anderson, G. Lester. "Quantitative Thinking as Developed Under Connectionist and Field Theories of Learning," *Learning Theory in School Situations*, p. 40. University of Minnesota Studies in Education, No. 2. Minneapolis: University of Minnesota Press, 1949.

A study containing important conclusions pertaining to methods to be used when teaching different types of students. Study points out that if skill in computation is taught the teacher can feel relatively free to choose her own methods. However if the ability to think mathematically is desired, an appropriate method must be selected.

2. Brownell, William A. and Moser, Harold E. "Meaningful vs. Mechanical Learning: A Study of Grade III Subtraction." Duke University Research Studies in Education, No. 8. Durham, North Carolina: Duke University Press, 1949.

A milestone on the path to a better understanding of an issue that has been with those teachers interested in arithmetic for many years. This study included evidence on how meaningful instruction affects the learning of subtraction in grade III.

3. Buswell, G. T. "Methods of Studying Pupil's Thinking in Arithmetic." *Arithmetic in 1949.* Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 55.

Gives six methods for studying pupil's think-

ing. Points out that further progress in the teaching of arithmetic is blocked because too little is known about the nature of meaning from the child's point of view.

4. Childhood Education. "Using What We Know About Children in Teaching Arithmetic." *Journal of Association for Childhood Education*. February, 1950.

Entire issue devoted to problems of teaching and learning arithmetic in the lower grades. Contains many leads and suggestions from which teachers may choose ideas for their own groups of pupils. Article by Mrs. Beatty on Re-Orienting to the Teaching of Arithmetic is particularly worthwhile for teachers in reappraising purpose and procedure.

5. Christofferson, H. C. "Division by a Fraction Made Meaningful." THE MATHEMATICS TEACHER, XLI (January, 1948), p. 32.

The author suggests several ways to present the division of fractions in a junior high school mathematics class.

6. Cronback, Lee J. "The Meanings of Problems." *Arithmetic in 1948*. Supplementary Educational Monographs, No. 66. Chicago: University of Chicago Press, 1948, p. 32.

An excellent treatment of meanings as related to problem-solving. Several illustrations of how certain children behave when confronted with problems. Also considers problem-solving as related to personality.

7. Cruickshank, William M. "Arithmetic Ability of Mentally Retarded Children: I. Ability to Differentiate Extraneous Materials from Needed Arithmetical Facts"; "II. Understanding Arithmetic Processes," *Journal of Educational Research*, XLII (November and December, 1948), pp. 161, 279.

A research article which throws light on the characteristics of the retarded child. Should be of special interest to those teachers who work with the slow-learning groups.

8. Glennon, Vincent J. "Testing Meanings in Arithmetic." *Arithmetic in 1949*. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 64.

Gives six causes for the lag in the development of adequate methods for measuring growth in understandings. Describes author's study of the growth and mastery of certain mathematical understandings on various levels.

9. Grossnickle, Foster E. "The Use of Multi-Sensory Aids in Developing Arithmetical Meanings," *Arithmetic in 1948*. Supplementary Educational Monographs, No. 66. Chicago: University of Chicago Press, 1948, p. 1.

A thorough discussion of three types of multi-sensory aids, namely; manipulative materials, films and filmstrips, and graphic representations. Sets forth a sequence of five steps in presenting a new process meaningfully.

10. Hartung, M. L. "Advances in the Teaching of Problem-Solving" *Arithmetic in 1948*. Supplementary educational Monographs, No. 66. Chicago: University of Chicago Press, 1948, p. 44.

Discusses several meanings of the term "problem-solving." Contrasts experience in reflective thinking provided by ordinary problems (the text-book variety) and "genuine problems."

11. Hartung, Maurice L. "Major Instructional Problems in Arithmetic in the Middle Grades." *Arithmetic in 1949*. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 80.

A masterful discussion of maximizing achievement in arithmetic when achievement is considered as a function of several variables. Points out that a modern arithmetic program requires a redistribution of emphasis and of time.

12. Johnson, J. T. "On the Nature of Problem-Solving in Arithmetic." *Journal of Educational Research*, XLIII (October, 1949), 110.

Gives evidence that supports hypothesis that vocabulary is one of the main ingredients of problem-solving obstacles in arithmetic. Notes that problems without numbers are harder than problems with numbers.

13. Johnson, J. T. "Are Ratios Fractions?" *Elementary School Journal*, XLVIII (March, 1948), 374.

Observes that the idea of ratio between two numbers needs much clarification and better teaching. Also makes the point that in spite of its elementary (and fundamental) nature, it is greatly neglected in the instructional program.

14. Koenker, Robert H. "Arithmetic Readiness for the Primary Grades." *Arithmetic in 1949*. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 26.

Sets forth a primary-grade program in arithmetic which should do much to eliminate the dislike for arithmetic that becomes such a problem in the intermediate grades.

15. Lazar, Nathan. "A Device for Teaching Concepts and Operations Relating to Integers and Fractions." *Arithmetic in 1949*. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 87.

The author presents the abacounter—a device consisting of a counting frame and a simplified abacus—as a device for making the concepts and operations of arithmetic real and meaningful.

16. Le Baron, Walter A. "A Study of Teacher's Opinions in Methods of Teaching Arithmetic in the Elementary Schools." *Journal of Educational Research*, XLII (September, 1949), 1.

Result of a questionnaire study indicates that mature and experienced teachers have formulated opinions which are about fifty per cent in agreement with current research findings and judgments of experts.

17. Morton, Robert Lee, "The Place of Arithmetic in Various Types of Elementary-School Curriculums." *Arithmetic in 1949*. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 1.

Establishes three criterion namely; the logical criterion, the social criterion, and the psychological criterion for determining the role arithmetic should occupy in the various types of curriculums.

18. Moser, Harold E. "Two Procedures for Estimating Quotient Figures when Dividing by Two-Place Numbers." *Elementary School Journal*, XLIX (May-June, 1949), 1949), 516.

Further consideration of the long division problem as presented by R. L. Morton and Worth J. Osburn (see *Elementary School Journal*, Vol. XLVIII, p. 141). This author contends that statistical studies over-simplify the problem faced by teachers of arithmetic.

19. Rambarter, Hazel K. and Johnson, Harry C. "Methods of Attack Used by 'Good' and 'Poor' Achievers in Attempting to Correct Errors in Six Types of Subtraction Involving Fractions." *Journal of Educational Research*, XLII (April, 1949), 594.

Another study of interest to those teachers investigating the problem of "poor" achievers in arithmetic.

20. Spitzer, Herbert F. "Procedures and Techniques for Evaluating the Outcomes of Instruction in Arithmetic." *Arithmetic in 1948*. Supplementary Educational Monographs, No. 66. Chicago: University of Chicago Press, 1948, p. 15.

This article discusses the kinds of evaluation instruments available at present with suggestions for improvement.

21. Sueltz, Ben A. "Important Issues in Arithmetic." THE MATHEMATICS TEACHER, XLII (April, 1949), 197.

The author lists about two dozen issues in the teaching of arithmetic relating to philosophy, objectives, the learning problem, evaluation and practices.

22. Swenson, Esther J. "Organization and Generalization as Factors in Learning, Transfer, and Retroactive Inhibition." *Learning Theory in School Situations*, p. 9. University of Minnesota Studies in Education, No. 2. Minneapolis: University of Minnesota Press, 1949.

A controlled experiment on the learning of the one hundred addition facts under three methods of instruction. Many conclusions of interest to the teacher of arithmetic, one of which states that the method in which generalizations play the dominant role produced the more favorable results.

23. Trimble, H. C. "Fractions are Ratios Too." *Elementary School Journal*, XLIX (January, 1949), 285.

A plea for a unified and comprehensive treatment of the ratio idea in the elementary and secondary schools. This article should be of special interest to intermediate grade and junior high school teachers.

24. Van Engen, H. "An Analysis of Meaning in Arithmetic." *Elementary School Journal*, XLIX (February and March, 1949), 321, 395.

Constructs a theory of meaning based upon the concept of operations—overt acts or possibility of actions—represented by symbols. Also discusses various "Theories" of meaning.

25. Wilburn, D. Banks. "Methods of Self-Instruction for Learning Arithmetic." *Arithmetic in 1949*. Supplementary Educational Monographs, No. 70. Chicago: University of Chicago Press, 1949, p. 35.

Discusses several experiments which show that pupils can learn arithmetic by methods of self-instruction.

26. Williams, Catharine M. "Arithmetic Learning in an Experience Curriculum." *Educational Research Bulletin*, XXVIII (September, 1949), 154.

Discusses several questions which usually arose in connection with author's study of arithmetic in an experience curriculum. Concludes that computation alone is not likely to develop mathematical insight.

27. Williams, Dolphus. "How Can We Improve Instruction and Achievement in Arithmetic?" THE MATHEMATICS TEACHER, (March, 1949), 156.

Advocates spending a least fifteen minutes at the blackboard drilling on fractions and whole numbers, "with no thought as yet of their application." The author presents a point of view which should be taken into consideration when developing a point of view toward instruction.

The Apple Theory

By WILLIAM R. RANSOM

Professor of Mathematics, Tufts College, Medford, Massachusetts

THE sum of $2a$ and $3a$ is certainly $5a$, but consider the reason usually given for it to beginning algebra classes. A fertile source of difficulties in later mathematics is the fact that so much is taught to the beginners about what algebraic work *looks like*, rather tahn what it actually *means*. One hears that a term is transposed and changes its sign, instead of that its negative is added to both sides of an equation. One hears of a fraction being inverted, parentheses being removed, and a factor being taken out from under a radical sign. And it is commonly said that $2a+3a=5a$ because two apples plus three apples makes five apples.

This explanation ignores the fact that not only an addition but also a multiplication is inovlved. "$5a$" is a product, and "a" is a *number*, not an apple. If $2a+3a=5a$ means that "two numbers plus three numbers makes 5 numbers," what is meant by "five numbers?" If $a=7$, does $5a$ mean "five sevens"? And does "five sevens" mean "77777", "7^5," or "$7+7+7+7+7$?" The "5 apples" is a *sum* (like the "$7+7+7+7+7$"); it is not a *product* like "$5a$": so the analogy fails at this point. And if instead of 2 and 3 as coefficients, we have x and y, or π and $\sqrt{2}$, the analogy fails utterly. How can we expect to build a sound knowledge of algebra upon the apple theory as an explanation of the distributive law?

So little light is thrown upon the true nature of $2a+3a=5a$ by the apple theory, that we find in a recent text (by an eminently reputable author) the statement that "only similar terms can be added!" If so, what becomes of the formula for the total surface of a cylinder: $\pi r^2+2\pi rh$? Here we have an addition just as truly as when the *form* of the sum is changed to $\pi r(r+2h)$, to $\pi(r^2+2rh)$, or to $\pi r^2(1+2h/r)$, etc.

The most important feature in algebra, next to its use of letters and negatives, is its revealing of advantageous changes in the *order of operations*. The distributive law as usually stated is slighted by those who teach beginners because it is commonly so stated that it does not reveal its utility and purpose clearly enough, and makes no explicit mention of *order*. Suppose it could be restated in a more striking form, as a principle about the possibility of interchanging multiplication and addition? If this were harped upon with beginners, to take the place of "looks-like rules" about similar terms, similar radicals, minus-before-parenthesis, adding fractions with common denominators, and some other special applications of the distributive law, the number of rules to be learned (and misapplied) might be greatly reduced. The following form has been tested for several years and seems to be successful:

If *two* numbers are first multiplied by a common factor and then added, the same result will be obtained as if they are first added and then multiplied—and vice versa: that is to say, if two numbers are first added and the sum is then multiplied, the same result will be obtained by first multiplying and then adding. (In this statement, "*two*" may be replaced by "three," "four," "five," . . . or by "two or more.")

This property of numbers is already familiar in elementary arithmetic. When we multiply twelve by three, we recognize that twelve is the *sum* of 10 and 2, and instead of considering the addition first and then multiplying by 3, we first multiply 10 and 2, each by 3, and then add the results, 30 and 6, in the customary abbreviated form, 36.

Of course "two apples three apples" is really a sum of two and three, and not a case of $2a+3a$ at all. To make a correct

application of $2a+3a=5a$ to the matter of the apples, we must first consider what happens in calculation with denominate numbers. First we make sure that the denominate numbers we add are expressed in the same units, and that the appropriate units are used in all rules involving products. Then the denominations are dropped, the calculations are carried out in pure numbers, and finally the appropriate name is affixed to the result. All this is ignored in the apple theory. And if we take "a" to represent one apple, we get a trivial case which reduces to $2+3=5$. We shall have to take "a" as a *number* of apples, say the number held by any one of a set of baskets. Then $2a$ is the number of apples in two baskets, $3a$ is the number of apples in 3 baskets, and $5a$ is the number of apples in $2+3$ or 5 baskets.

The apple theory loses all its obviousness as soon as the 2, the 3, or the "a" cease to be small integers. So let us leave the apples out of it and explain $ba+ca=(b+c)a$ as what it is, a reversal of order in the operations of addition and multiplication.

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BOOK REVIEWS

Mathematics Dictionary. Glenn James and Robert C. James, with the assistance of A. A. Alchian, E. F. Beckenbach, Clifford Bell, H. V. Craig, A. D. Michal, and I. S. Sokolnikoff. Revised and enlarged edition. v+432 pp. D. Van Nostrand Co. Inc., 1949. Price, $7.50.

To the original edition which appeared in 1942, the authors have included terms from metric differential geometry, theory of functions, advanced calculus, differential equations, theory of groups, theory of matrices, theory of summability, point-set topology, general analysis (including integral equations and the calculus of variations), analytic mechanics, the theory of potential, and statistics. The purpose of the dictionary is "to make it useful for students, engineers, and others using mathematics in their profession."

That the authors have succeeded in their objective is due in large measure to the hybrid production of a dictionary and an encyclopaedia. The terms are not only defined explicitly (which in most cases would give very little insight to the reader) but also described, illustrated by example, and frequently exhibited by a diagram. From abacus to Zeta function one can readily obtain information that will recall concepts forgotten through disuse. Frequently, one can go from a more abstract to a simpler concept to gain sufficient information for applying the term considered. Thus one can go from continuum to compact (sets) to limit point and gain sufficient understanding of the term continuum. It cannot be expected that in such a volume all concepts will be covered. For example, *norm* of a complex number and acceptance sampling, were two terms picked at random by the reviewer which were not included. No doubt many other terms will be found missing.

The format of the dictionary is excellent. Major terms are in capitalized bold-face type and sub-topics in small bold-faced type. The spacing permits easy reading; there is no feeling of ordinary dictionary compactness and dryness. The book is well bound, and the tables given in the appendix indicate a discriminating and difficult selection on the part of the editors.

The book should prove especially valuable to high school teachers of mathematics. Here will be found the answer to any collegiate mathematical terms about which their students may inquire. Here they may clear up some hazy concepts concerning elementary mathematics which they teach. Here they may clarify their use of mathematical terms in writing, teaching, or doing research. The book should be in every school, college, and industrial library.—HOWARD F. FEHR.

Introduction to Mathematics: A Survey Emphasizing Mathematical Ideas and Their Relations to Other Fields of Knowledge. By Hollis R. Cooley, David Gans, Morris Kline, and Howard E. Wahlert. Second Edition. Boston, Houghton Mifflin Company, 1949. xxii+710 pages, $4.25.

The revision of a book so favorably received since its first publication twelve years ago has resulted in a most admirable edition. The authors' objective is to have the student achieve some degree of understanding of mathematics and of its role in the advance of civilization. It is contended that understanding may be achieved through an intuitive approach and by a minimum of practice on technical devices and manipulations. The reviewer believes that this contention is wholly justified. Much use is made of the experience of the student in the presentation and development of topics. Further meaning is attained through a multitude of applications of mathematics to the natural sciences, the social sciences, the arts, and philosophy. There will be very few students who find this type of mathematics a "meaningless grind."

The format of the book has been improved; diagrams are attractive and lucid. There are more manipulative exercises included; their use should vary with the individual student. As with the first edition, the end-of-chapter references are detailed and may be readily integrated with the textual material to make for both a more extensive and more intensive treatment. Complete bibliographical citations for all references are included in a bibliography. Supplementary material for those teachers wishing a more detailed treatment of trigonometry is available in an appendix.

This text is heartily recommended for a first-year college course in non-specialist mathematics. It may also serve the student-specialist as an integrated overview of mathematics. The reviewer has successfully used portions of the first edition as collateral reading for his high-school classes. The book has much to offer in the way of enrichment material to the high-school teacher. It fills an important place in his library.

A list of contents follows: reasoning and mathematics; numbers and their use; arithmetic in retrospect; algebra as generalized arithmetic; algebraic equations; exponents and scientific notation; exponents and logarithms; elementary geometry in retrospect; trigonometry; co-ordinate geometry; the function concept; algebraic functions; progressions and the mathematics of investment; exponential and logarithmic functions; the trigonometric functions and

their application to sound, electricity, radio, and light; linear and exponential trends; descriptive statistics; probability, and the need for caution in the use of functions; the rate of change of a function; the limit of a sum; infinite classes; non-Euclidean geometry; theory of relativity; the nature of mathematics; mathematics and modern civilization.—MAX BEBERMAN.

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Analytic Geometry. By Kells and Stotz. Prentice-Hall Inc., New York, 1949. 280 pages. Price, $2.85.

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To achieve this aim, important techniques are introduced early in the study and used repetitively. Thus, vectors are discussed at first opportunity; polar coordinates prior to conic sections and later with each type of conic section, special curves, asymptotes, and cylindrical coordinates; graphing prior to more difficult ideas in order to give students an insight into difficult concepts. Individual chapters deal comprehensively yet in elementary fashion with basic concepts, graphs of functions, fundamental formulae, straight line, circle, polar coordinates, parametric equations, conic sections, general equation of the second degree, miscellaneous curves, curves of empirical data, graphs and loci in space, and cylindrical, spherical and polar coordinates.

Two hundred twenty five (225) illustrations in conjunction with one thousand three hundred fifty (1350) problems (odd numbered answers supplied at rear of text) and large type print combine to make the study easily pursued and understood.

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fundamental concepts, the plane, straight line, space loci, and the quadric surface. Though there exists a division in the arrangement of two- and three-dimensional study, the presentation is such that the reader may readily integrate these concepts.

In addition there is included by topics a quick reference table to some eighteen standard textbooks for those desirous of further reference in their study. Similarly, there are included two appendices wherein there are offered sample examinations (with answers) and tables of squares, cubes, roots, common logarithms, natural logarithms, natural trigonometric functions, and values of e^x and e^{-x}.

Throughout the outline many proofs of theorems exist as do some two hundred accurately drawn figures and illustrations, and typical problems and exercises (with answers).

Aside from minor differences pertaining to definitions and certain omissions, this treatment of Analytic Geometry is recommended *as an outline* for purposes of review and self instruction.—EDWARD J. THOMAS.

The Geometry of the Zeros of a Polynomial in a Complex Variable. By Morris Marden. New York, American Mathematical Society, 1949, 10+183 pp. $5.00.

This is number three of the series of Mathematical Surveys of the American Mathematical Society. It is an attempt to "bring together results at present scattered throughout the mathematical journals and to unify and simplify both the results and the method of treatment."

The preface contains a statement of the problem. The zeros of a polynomial $f(z)$ are functions of various parameters, usually coefficients. If these parameters are allowed to vary within prescribed regions in the complex plane, the corresponding locus R of the zeros of $f(z)$ is to be determined. If this is not feasible, it may be possible to specify a region S containing R.

The work was written for, and is of unquestionable value to the "prospective specialist" as well as the "applied mathematician and engineer" and the "general mathematical reader" who wishes to be introduced to "some relatively new, interesting, and significant material." The mathematical maturity necessary to the understanding of this book is beyond that of most high school teachers of mathematics, but this is to be expected from the nature of the material and the audience for which it was written.

A particularly complete bibliography is included which is keyed both ways. That is, indications are given in the text to the pertinent items in the bibliography and with each item of the bibliography is a reference to those paragraphs of the text which contain related material.—FRANK HAWTHORNE.

Conference—Mathematics for the Elementary Grades
State College for Teachers, Buffalo, N.Y.
July 27 and 28, 1950

July 27 (Thursday)

10:00 A.M.—Prof. Wm. A. Brownell, Northwestern U.
"What is Arithmetic?"

1:30 P.M.—Prof. Brownell
"Making Arithmetic Sensible and Useful"

July 28 (Friday)

Group meetings to be determined by interests and problems.

AIDS TO TEACHING

HENRY W. SYER
School of Education
Boston University
Boston Massachusetts

DONOVAN A. JOHNSON
College of Education
University of Minnesota
Minneapolis, Minnesota

BOOKLETS

B.31—Mental Multiplication

Charles Lipkin, 249 West 34th Street, New York, N. Y.

Pamphlet; 24 pages; 5″×8″; 25 cents

Description: This pamphlet gives a variety of methods of multiplying mentally without the use of paper and pencil except for purposes of recording answers. Methods are given for the mental multiplication of any number regardless of the number of digits by any other number. Problems are worked out step by step to show the mental process in multiplication and to compare the mental method with the usual method. Four different methods are illustrated depending on the complexity of the problem. Considerable space is used to sell the reader on the advantage of mental multiplication and how it can be used to make machine calculations more efficient.

Appraisal: This pamphlet is issued by the author to promote the sale of the fourth edition of the book *Mental Multiplications*. It is written in the first person expressing the views of the author regarding the best way to multiply mentally and his experiences with computing machines. It contains a very limited amount of material that can be taught to immature minds. Its real value might be in the stimulation of rapid mental computation by the very superior student.

CHARTS

C.14—Astronomical Geography Chart (8211)

Denoyer-Geppert and Co., 5235–5237 Ravenswood Avenue, Chicago 40, Illinois 36″×24″; $16.50.

Description: This chart is beautifully lithographed on heavy cloth-lined stock. It is mounted on $\frac{1}{4}$″ board and further reinforced with a metal trim. It is of exceptionally sturdy build. It contains a chart of the heavens, solar system, change of seasons, phases of the moon, and many more astronomical phenomena.

Appraisal: This chart enables the teacher to explain such everyday phenomena as cause of day and night, variation in length of day and night, change of season, phases of the moon, eclipses, etc. By means of colored transparencies and movable discs through which light shines when the chart is placed before a window, the relations of sun, moon, and earth are brought out. The planets are represented, placed in relative order with respect to distance from the sun. In the center of the chart is an effective representation of the constellations, which can be so moved as to show their position in the sky at any hour of any night of the year. Somewhat expensive for a chart for secondary-school use. (Reviewed by Jacqueline LeMay, Boston University, Boston, Massachusetts)

EQUIPMENT

E.27—Fractions Made Easy

Ideal School Supply Company, Chicago, Illinois

Price $0.35; per dozen $3.50

Description: The twelve white 4×4 basic cards consist of one blank card and

eleven that are regularly marked off into parts varying from halves to sixteenths. Green cards have been cut into parts which will fit on those that are marked on the basic white cards. Three printed problem cards are provided which direct pupils to carry out certain manipulations. For instance, one part of the fourth problem on the first card instructs pupils to "Cover $\frac{3}{4}$ of C, D and Y" (C is a white card divided into fourths by four horizontal lines, the D card is divided into eights by four horizontal lines and a bisecting vertical line and the Y card is divided into fourths by two diagonal lines).

Appraisal: This device makes available commercially a kind of material that many good teachers have had pupils cut out for themselves. The principle of having pupils have a lot of experiences with various parts and the interchanging of these parts is good, but several improvements could be suggested to make the *Fractions Made Easy* material of maximum benefit: 1. the directions should describe developmental experiences with these fractional parts that should be carried out prior to the use of the problem cards for many teachers will start using these problem cards too early, 2. more help should be provided as to the procedure for transferring from the objective stage to the symbolic stage, 3. the parts would be more easily fitted together if they were constructed out of heavier materials, 4. the development of the concept of parts could be more completely provided for if the basic cards and parts included some that were more irregular, 5. for quick identification and for emphasizing relationships, there would be value in having the halves, fourths, eights and sixteenths one color and the remaining parts another color, and 6. the teachers should be given more help in the way these parts can be used to help pupils understand the addition, subtraction, multiplication and division of fractions.

These sets of parts would be valuable to have in the hands of each pupil or else have them available on a number table. The teacher who uses them should recognize that they can be useful for several purposes including development of concepts and procedures, drill, relearning experiences, and as materials which children can use to prove their answers. (Reviewed by Robert L. Burch, Boston University, Boston, Massachusetts)

E.28—Add-O; E.29—Count-O; E. 30—Mult-O

Benjamin H. Sanborn and Co., 221 East 20th Street, Chicago 16, Illinois
Games; $2.00 plus mailing charges for each game

Description of E. 28–30: The game Count-O consists of large cards with a four-by-four number chart on each side and the small cards contain a number less than 100 on each side. The purpose of the game is to teach the child to recognize and read numbers both in and out of sequence. One series presents the numbers from 0 to 50 and a second series from 50 to 100. It is played in a manner very similar to bingo. Each child is given a large card and several beans. A small card is drawn, the number read, and a bean placed on the corresponding number on the pupil's chart. The game proceeds until some pupil covers four numbers in a vertical or horizontal row. The check requires the winning pupil to read his numbers.

The game Add-O is played in the same manner. The small cards contain the 100 number combinations from 0 to 9 and the pupil cards contain the same number combinations written vertically as well as horizontally. Similarly the game Mult-O presents the 100 multiplication combinations of one-digit numbers.

Appraisal of E.28–30: These card games provide a variation of the usual drill techniques. Besides furnishing motivation through play, individual differences are provided for by making it possible for every child to participate actively and for each child to have an equal chance to win. The games should prevent the practice of incorrect responses since each card gives

the correct answer. There are several variations of the games and the games may also be played without the direct participation of the teacher. The cards are printed in black on white on durable material. The teacher or school lacking funds for these games can have pupils make similar games as a class project.

FILMS

F.49—Multiplying Fractions

Knowledge Builders, 625 Madison, New York

16 mm. sound film; 1 reel; black and white; $40.00

Description: A model airplane, built to the scale $\frac{1}{8}'' - 1'$, is the basis for an explanation of the multiplication problem $\frac{1}{8} \times 7$. A rule for the multiplication of fractions is derived and applied to a kitchen problem on the increase of a recipe. An apple and a block divided into parts are used to show that multiplication is inferred by the use of the word "of." The denominator of fractions is shown to be similar to units of measure, for example $3/10 \times 5$ is related to 3 oz. $\times 5$. Several problems are worked to show the application of the multiplication rule.

Appraisal: The film spends considerable time working problems, developing a rule of computation, and showing a shortcut in the typical classroom manner. It would seem that a 10 minute film could best be used to build an understanding of a process and leave practice, rules, and shortcuts for classroom discovery and discussion. This film does illustrate the meaning of fractions and multiplication of fractions by illustrations that are appropriate. However, the commentary and photography often lack interest.

F.50—Latitude and Longitude

United World Films, 1445 Park Avenue, New York 29, New York

Educational collaborator: Margaret Simpson. January, 1948; 16 mm. film, black and white; sound; 320 feet; 9 minutes; $45.00

Description: Except for a few nautical scenes, the entire film is done in animation, By opening the earth globe and exposing the interior to view, the film vividly shows that the latitude of a place is the angular distance north or south of the equator. Other items clearly explained are that all places at the same angular distance from the equator lie on a circle parallel to the equator; latitude angle is independent of the size of the globe; the greatest latitude a place can have is ninety degrees north or ninety degrees south; a latitude reading only will not fix a position. The discussion of longitude opens with the drawing of the Greenwich meridian on a globe showing the outline of the continents. Longitude is defined as the angular distance of a place east or west of the Greenwich meridian. Again, the dissection of the globe makes this meaning very clear. The film goes on to develop the ideas that meridians of longitude are semicircles thru the poles at right angles to the equator; a meridian line is generated by connecting points at equal angular distances from the Greenwich meridian; the greatest longitude a place can have is 180 degrees; a longitude reading only will not fix a position. By intersecting circles it is then shown that only one place on the earth will have a given latitude and a given longitude.

Appraisal: The film was not produced for mathematics classes and makes no special mention of the mathematics involved. However, pupils who have had plane and solid geometry readily see and appreciate the applications of many geometric principles. This is especially true of the theorems about the sphere. It could be used to advantage with plane geometry classes after the study of locus and of angle measure. Solid geometry classes like to see it before and again after the work on spheres. Use of the film clarifies the concept of latitude and longitude even with

mature pupils. Students enjoy trying to find the mathematics involved without having it specifically pointed out to them.

Animation and sound are excellent. Commentary is simple and clear. (Reviewed by Frances Burns, Oneida High School, Oneida, New York)

FILMSTRIPS

FS.67-History of Our Number System

Young America Films, Inc., 18 East 41st Street, New York 17, New York
Educational collaborator: Henry W. Syer; November, 1949; 35 mm. filmstrip; black and white; silent; 35 frames; $3.50

Description: The topic is introduced by indicating that numbers are symbols that tell "how much" or "how many." Several frames show that early man had no need for a number system such as we have today, but could use his fingers or sticks or pebbles to show "how many." Picture writing and the single stroke are then shown as early methods of recording a count. Babylonian cuneiform numerals, Egyptian hieroglyphic numerals, Hindu and Roman numerals are pictures to convey the idea that in early times groups of single strokes represented small numbers, but that for the number ten a symbol was used. The use, by these different peoples, of this symbol for ten plus the mark for one to write the number eleven is shown and the reason for this practice is mentioned. The probable evolution of the Roman V and of our present symbols for 2 and 3 from groups of single strokes is shown. The Greek and Hebrew use of letters of the alphabet for numerals is mentioned. The growth of the idea of place value is pictured by showing the abacus and how it developed from pebble counting. The introduction of the zero as a place holder for written numbers is mentioned. The advantage of operating with Hindu-Arabic numbers over the Roman numerals is also pictured. A frame showing several common mathematical symbols and a notation of the dependence of modern life on our number system completes the filmstrip.

Appraisal: The filmstrip is excellent for historical background and for creating interest in the origin of our number system. It is best suited to junior high school classes but older pupils, who are not already familiar with the material will enjoy it. It is a definite aid to the teacher presenting these facts for the pictures and the drawings necessary to illustrate the ideas could be neither quickly nor easily made by the instructor. The photography is excellent. The frames carry a sufficient but not confusing amount of detail. Captions are clear and simple. (Reviewed by Frances Burns, Oneida High School, Oneida, New York)

FS.68—History of Linear Measure

Young America Films, Inc., 18 East 41st Street, New York 17, New York
Educational collaborator: Henry W. Syer; November, 1949; 35 mm. filmstrip; black and white; silent; 36 frames; $3.50

Description: Direct comparison of objects is discussed as the earliest way of making one object equal in length to another. Progress from this method to that of using some part of the body as a unit is shown. Pictured are the cubit, span, palm, foot, and digit measures. The units of "stone's throw," and "arrow's flight," "day's journey" for measuring longer distances are mentioned. The confusion and inaccuracy resulting from unstandardized measures is pointed out. Origins of the Roman foot, mile and inch are given. The methods by which the English yard and the inch became standardized are shown. The meter and its relationship to the yard is briefly covered. Also noted are the general acceptance of the meter by most nations; the comparatively recent adoption of standards in the United States; the dependence of modern industry on these exact standards of measure.

Appraisal: The filmstrip gives an interesting and historically accurate account

of a topic for which it is difficult to find good pictorial material elsewhere. The illustrations and captions for the frames are well chosen and the scenes well drawn. Although the time span is from prehistoric to modern, a confusing amount of detail is avoided. While probably intended primarily for junior high school students, the filmstrip could be used to advantage with older pupils who are not already familiar with these facts. It could also serve as motivation for further study especially of the metric system and of some of the less well known units of measure. This is the kind of information which can be taught better with visual materials of the filmstrip type than the teacher alone could do it. (Reviewed by Frances Burns, Oneida High School, Oneida, New York)

INSTRUMENTS

I.19—The New Gravity Protractor (No. 130)

Yoder Instruments, East Palestine, Ohio
24″ long; $3.50

Description: The versatile hardwood instrument is a twenty-four-inch ruler with an attached semicircular gravity protractor. Angles are drawn by holding the instrument in such a position that a pendulum points to the desired graduation on the circumference of the protractor; the pendulum swings freely on roller bearings. The protractor is graduated in half-inches and the inches are numbered in either direction from the rule's center. The large graduations and numbers enable students to see the dimensions of diagrams from their seats. Three rubber tips on the reverse side of the tool prevent slipping and minimize the smudging of lines during drawing. Total weight of the device is $1\frac{1}{4}$ pounds. It is easy to manipulate both because of its light weight and a round knob which serves as a convenient handle.

Appraisal: The instrument is desirable for blackboard drawing because it obviates the necessity of employing an individual protractor, triangle, T-square, and straightedge. This device may serve all of these functions. The price of the combination rule and protractor is high; yet it is less than the total cost of most blackboard sets whose tools serve no functions not performed by this single instrument. (Reviewed by Bernard Singer, Hyannis, Massachusetts)

MODELS

M.12—Tetrahedron Set

Mathaids Co., 609 Almond Street, Syracuse, New York
$3.00 per set

Description: This set consists of (a tetrahedron made) $\frac{1}{8}$″ bronze rods each 12″ in length, and also 3 extra rods to demonstrate altitudes of faces and of the solid. The corners are fastened together by brazing. At the midpoints of the edges, hooks are securely fastened so that four rubber bands may be stretched between sets of 3 points, and a regular octahedron will be clearly shown.

Appraisal: The tetrahedron is indeed difficult to demonstrate on the blackboard. With this aid, the tetrahedron set, the teacher will save not only time but also will explain the properties of the figure and of the octahedron in a more efficient manner. In the field of chemistry this would be especially of value to explain the valence of the carbon atom. The students cannot help but understand the relations. (Reviewed by Jacqueline LeMay, Boston University, Boston, Massachusetts)

SOURCES OF MATERIALS FOR LABORATORY WORK

SL.17—Grove's Grid-grafe

Yoder Instrument Co., East Palestine, Ohio; 70¢ per packet

Description: Grove's Grid-grafe system can be used in mathematics for similar figures, plane and solid; congruent figures; scale drawing; ratios; illustrate fractions and decimals. The system consists of three transparent grids. Nos. 1, 2, 3, and three graph sheets, Nos. 1, 2, 3. Any picture or

illustration can be accurately transferred from any book, magazine, etc., by placing a grid over the picture or illustration and with the aid of the cross lines of the grid, drawing the picture on the graph sheets. For example, if No. 2 grid is used on No. 2 graph paper the reproduction will be identical with the original picture (a one to one correspondence of like dimensions, areas, and volumes). If No. 2 grid is used on No. 3 paper, the reproduction will be an enlargement of 2 to one in like dimensions and 4 to 1 in areas and 8 to 1 in volumes. If No. 2 grid is used on No. 1 paper, the reproduction will be a reduction of 1 to 2 in like dimensions, 1 to 4 in areas, and 1 to 8 in volumes. These and other combinations of grids and graph paper and their results are tabulated in the following table:

	Graph paper No. 1 $\frac{1}{8}$" sq.	Graph paper No. 2 $\frac{1}{4}$" sq.	Graph paper No. 3 $\frac{1}{2}$" sq.
Grid No. 1	L 1 to 1 A 1 to 1 V 1 to 1	L 2 to 1 e A 4 to 1 e V 8 to 1 e	L 4 to 1 e A 16 to 1 e V 64 to 1 e
$\frac{1}{8}$" squares			
Grid No. 2	L 1 to 2 r A 1 to 4 r V 1 to 8 r	L 1 to 1 A 1 to 1 V 1 to 1	L 2 to 1 e A 4 to 1 e V 8 to 1 e
$\frac{1}{4}$" squares			
Grid No. 3	L 1 to 4 r A 1 to 16 r V 1 to 64 r	L 1 to 2 r A 1 to 4 r V 1 to 8 r	L 1 to 1 A 1 to 1 V 1 to 1
$\frac{1}{2}$" squares			

L—linear; A—area; V—volume; e—enlargement; r—reduction

Appraisal: This valuable teaching aid is essential in subjects such as mathematics to teach form, perspective, proportion and line relations. It is simply done and stimulates the interest of the student because he draws accurately from the beginning. In most classes few can do accurate drawings. This will provide the students with actual reproductions from which he can learn much more than from a drawing which someone else has made. This material is very inexpensive and is worth trying. (Reviewed by Jacqueline LeMay, Boston University, Boston, Massachusetts.)

SL.18—Manual and Instructions for the "Schacht" Dynamic Geometry Instruments

W. M. Welch Scientific Company, 1515 Sedgwick Street, Chicago, Illinois. Booklet; $7\frac{3}{4}" \times 10\frac{1}{2}"$; 46 pages; 50¢ each

Description: After fifteen pages of introduction and general description of the six Schacht devices (triangle with constant midpoints, adjustable triangle, quadrilateral device, circle device, parallel line device, and criteria quadrilateral) there are twenty-eight pages of sample "guide sheets" to show the type of classroom materials which can be put by the teacher into the hands of the pupils to lead them, through proper manipulation of the devices, to the discovery of geometric generalizations. These guide sheets are in workbook form, with questions followed by blank spaces for answers, and tables ready for the recording of experimental data. In general the sheets are divided into the following sections: the problem, the device, the figure, the introduction, the procedure, the data table, the observations, the hypotheses, deductive proofs and additional problems.

Appraisal: Although the most effective combination would naturally be this booklet with the models for which it was written, there is usefulness in the booklet alone. It shows the types of questions, the method of procedure, the tables which can be supplied to the pupils and the degree of

freedom which they can be expected to be entrusted with. There is much to be learned about the laboratory approach to plane geometry by reading through this booklet.

Its appearance is adequate, but not outstanding. Many will feel that the workbook section should have printed separately from the suggestions to teachers. It is not mentioned whether teachers are free to reproduce the guide sheets supplied, but since the booklet is copyrighted, it is assumed that they are not allowed to. Would such sheets, printed in an inexpensive manner, not be more helpful if sold in quantity? Is there not a possibility that such sheets, printed in "ditto" ink, with permission to reproduce in any quantity, would encourage more widespread use of this material?

Correction

The Diagrams in Three Dimensions for Solid Geometry, made by The Orthovis Company, reviewed in this column in the January issue, may be ordered from Newson and Company, 72 Fifth Avenue, New York 11, N. Y.

New Mathematics Kit

Can your students draw an original straight line? Can you draw such a line? Euclid postulated the straight line, but he did not have any way to draw one. In fact it was less than one hundred years ago when the first theoretically perfect means of tracing a continuous straight line in a plane was discovered.

This interesting problem of the straight line is the subject of a new mathematics kit produced jointly by Science Service and The National Council of Teachers of Mathematics. The kit contains materials for assembling four workable models of linkages which trace straight lines and other curves. An explanatory leaflet is included. This kit is known as Straight Line Unit, No. 113. A supply of the kits has been secured for members of the Council.

Also a few sets of the previous unit, Geometric Models Unit, No. 102, are still available. The price is 50c each or 3 for $1.00. Please include payment with each order. All kits should be ordered from

Professor M. H. Ahrendt
Anderson College
ANDERSON, INDIANA

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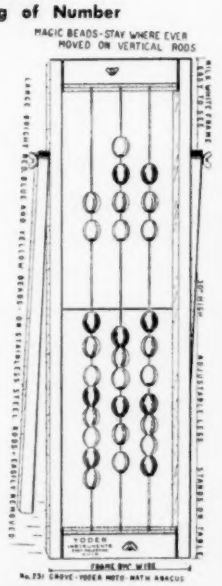

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For Your Library

MATHEMATICS DICTIONARY, *James and James.* Revised and Enlarged Edition. This is a comprehensive dictionary of terms beginning with arithmetic and extending through calculus. It is a real condensation of mathematical concepts arranged in time-saving alphabetical order. Contains advanced material in physics, chemistry, and engineering as well as simple terms. $7.50

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